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Anna D'Annunzio, Antonio Russo, Shiva Shekhar

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Abstract

The adtech industry plays a key role in facilitating connections between digital publishers and advertisers. This paper studies the impact of vertical integration between an intermediary and a major publisher on the online advertising ecosystem and the provision of content. We find that vertical integration enables the intermediary to leverage exclusive access to data, leading to dominance in the intermediation market. As a result, the integrated intermediary is able to collect a larger ad-tech tax from independent publishers by shading its bid for impressions. This practice reduces investments in content by independent publishers, while the integrated publisher increases its investment. Therefore, the net effect of vertical integration on consumer welfare and total welfare can be positive or negative. We discuss potential policy interventions that restore the outcome as under vertical separation.

JEL-Codes: D430, D620, L820, M370.

Keywords: online advertising, intermediaries, vertical integration, adtech tax, content quality.

Anna D'Annunzio
Toulouse Business School / France
dannunzio.anna@gmail.com

Antonio Russo
Institut Mines-Telecom Business School
Evry / France
antonio.russo@imt-bs.eu

Shiva Shekhar
Tilburg School of Economics and Management
Tilburg / The Netherlands
shiva.shekhar.g@gmail.com or
s.shekhar_1@tilburguniversity.edu

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1 Introduction

Online advertising is the main source of revenue for many digital publishers, including news and review websites, blogs and app developers. In this market, a complex chain of intermediaries, called the adtech industry, connects advertisers with publishers.[1] This industry absorbs a substantial share of the money advertisers spend to place ads on digital publishers, in what is typically referred to as the *adtech tax*.[2]

In this paper, we focus on two issues that have drawn the attention of regulators and practitioners regarding the adtech industry: its high level of concentration and the integration of its main players in content provision (ACCC, 2019; CMA, 2020; Stigler Committee on Digital Platforms, 2019). The main operator in this industry is Google, which dominates almost every link of the adtech chain (IAB, 2017). Moreover, its adtech services are part of an ecosystem that includes multiple content and consumer services such as digital maps (Google Maps), email (Gmail) and video streaming (YouTube). These aspects raise several mutually reinforcing concerns. First, an integrated firm may require advertisers to use its own intermediation services to access the ad inventory on its ecosystem (as Google currently does). As a result, the firm can not only control access to this inventory as a monopolist, but also have unique access to user data from its own websites. This data may allow the integrated firm to more effectively allocate ads (e.g., more effective frequency management or precise targeting), not only when managing its own ad inventory, but also that of third-party publishers as an intermediary. These competitive advantages vis-à-vis other intermediaries, combined with the adtech industry's general opacity, may enable the integrated firm to extract a large adtech tax from advertisers and publishers.[3]

A second concern pertains to the relation between the adtech and content industries (Cairncross, 2019). If the integrated firm can capture a significant share of the ad revenue of third-party publishers, their incentives to invest in content may decrease, contrary to

[1]This chain includes supply-side platforms (SSPs) that collect ad inventories from publishers and run ad auctions; demand-side platforms (DSPs) that allow advertisers to buy ad inventories; publisher ad servers, that manage publishers' inventory and decide which ad to serve, based on the bids received from SSPs and direct deals between the publisher and advertisers. See CMA (2020) for an analysis of this market.

[2]The ISBA and PwC estimate that roughly half of the value bid by advertisers in programmatic advertising auctions actually reaches the publishers carrying their impressions, see https://tinyurl.com/4995vd4w. The Competition and Markets Authority's recent study of the digital ad market estimates that at least 35% of the value of advertising bought is captured by intermediary fees (CMA, 2020).

[3]In 2023, the U.S. justice Department sued Google for illegally monopolizing the digital advertising market (see https://tinyurl.com/mvy8r2d8). The European Commission has also recently raised concerns regarding Google's dominant position in the ad intermediation market, see https://tinyurl.com/3f748hzh.Furthermore, some U.S. States sued Google in 2020 for illegally monopolizing the digital advertising market, see https://casetext.com/case/texas-v-google-llc. Most recently, Google was the subject of a lawsuit launched by several media firms, claiming that "without Google's abuse of its dominant position, the media companies would have received significantly higher revenues from advertising and paid lower fees for ad tech services". See https://tinyurl.com/y2acf7e5.

the incentives of the integrated publisher. Hence, by taking advantage of its ecosystem, the integrated firm can reinforce its position in the market for content. In a feedback loop, the firm may strengthen its position in the market for intermediation as well.

To address the above concerns, this paper studies the digital advertising and content markets, analysing the interplay between integration in adtech, access to consumer data and investment in content quality. The analysis also explores the connection between the adtech tax and the integration of dominant adtech firms in content provision within ecosystems.

We build a model with two publishers and two ad intermediaries, comparing the case where intermediaries and publishers are separate to that where one intermediary is integrated with a publisher. In the baseline model, consumers can single-home or multi-home and advertisers have diminishing returns to advertising to the same consumer.[4] Each visit to a publisher by a consumer generates an opportunity for an ad impression. In the vertical separation scenario, the publisher sends a bid request for this impression to both intermediaries, that collect advertiser bids via first-price auctions. Having received those bids, the intermediaries bid to the publisher in turn to acquire the right to distribute the impression. The publishers have no visibility on the bids placed by the advertisers. Under vertical integration, the integrated intermediary has exclusive access to the ad inventory of the subsidiary publisher and to the information generated by consumers on that website. Hence, only the independent publisher sends bid requests to both intermediaries.

Under vertical separation, both intermediaries track consumers across both publishers. However, they cannot observe which other ads (if any) a consumer has been exposed to when these are distributed by the other intermediary.[5] Therefore, the advertisers perceive the impressions on multi-homers as substitutes when auctioned by different intermediaries. Hence, in equilibrium each advertiser buys ads from either one intermediary or the other. The intermediaries' ability to control the frequency of impressions maximises the advertisers' willingness to pay for each impression, and competition among advertisers ensures that all of it is extracted by the intermediaries. However, the intermediaries compete as well to acquire the publishers' impressions. In equilibrium, both publishers receive the full revenue generated from the respective ad inventories: the intermediaries do not extract any adtech tax.

Things work differently in the vertical integration scenario. Unlike its rival, the integrated intermediary can observe which consumers multi-home and to which impression

[4] Decreasing returns in ad exposure imply that advertisers value the ability to manage the number of times a user is shown an ad over a period of time (CMA, 2020).

[5] This assumption captures the difficulty of managing frequency of exposure when advertisers use multiple demand-side platforms. Google's own campaign evaluation tools emphasize unique users and impression repetition (https://support.google.com/google-ads/answer/2472714?hl=en&ref_topic=3123050).

they are exposed to on the integrated publisher. Thus, due to the potentially wasteful repetition of ads, the advertisers discount their bids for impressions auctioned by the non-integrated intermediary. As a result, the integrated intermediary faces weaker competition when bidding for the impressions generated by the independent publisher. In equilibrium, it extracts the maximum willingness to pay for these impressions from the advertisers, but pays only a fraction of this revenue to the publisher. That is, integration enables the firm to extract an adtech tax by "shading" its bid. This tax increases with the share of multi-homing consumers because the informational advantage of the integrated intermediary stems precisely from tracking consumers across different websites.

Our analysis explores how the firms' incentive to manipulate the adtech tax drives their investment in content quality. Providing more content quality increases the share of multi-homers, hence the adtech tax. As a consequence, the independent publisher invests less than in the separation scenario, because higher investment raises the adtech tax. The effect of integration on investment by the integrated publisher is opposite. By the same mechanism, the integrated publisher has higher incentives to invest in quality.

Although vertical integration changes the distribution of profits in favor of the integrated firm, the effects on consumer surplus and welfare are a priori unclear. This is because quality is under-provided under vertical separation, as each publisher only internalizes the effect of quality on advertising revenue, ignoring consumer surplus. Integration aggravates the under-provision for the independent publisher, but alleviates it for the integrated firm. Which effect prevails depends on the distribution of consumer preferences (see Section 5).

In the final part of the analysis, we consider how a regulator can correct the market distortions caused by integration. Possible measures include prohibiting foreclosure from the impressions on the integrated publisher and removing the exclusive control on the data generated when consumers visit such publisher.[6] We show that neither measure would work by itself. Our analysis highlights a trade-off between the efficiency of the ad market and consumer privacy. This observation is consistent with the ongoing debate on the possible unintended consequences of privacy regulation such as the GDPR. Furthermore, the analysis suggests that ecosystems such as Google may support the adoption of stricter privacy rules to preserve their competitive advantage. Such rules, limiting data access to multiple intermediaries, can result in a higher adtech tax, to the detriment of third-party publishers and content quality.

The remainder of the paper is structured as follows. In section 2, we discuss the related literature. In section 3, we present the model setup and then in Section 4, we present the analysis for the vertical separation case and the vertical integration case. In Section 5 we derive the welfare effects of vertical integration. In Section 6, we study some

[6] This is consistent with the provision in the DMA to "prevent gatekeepers from unfairly benefiting from their dual role" and on the discussion on interoperability to boost contestability of markets.

policy interventions. We conclude in Section 7. The proofs are available in the Appendix.

2 Related Literature

This work contributes to the recent literature on the impact of intermediaries in the online advertising market. Sayedi (2018) studies publishers' budget allocation among real-time bidding and reservation contracts. D'Annunzio and Russo (2020) model a monopolist ad network with multi-homing consumers and advertisers, studying the effects of the ad intermediation on the publishers' advertising intensities. Marotta et al. (2022) considers a platform that shares consumer information with advertisers and its affects competition on the product market. The above cited works consider a monopolist intermediary in the advertising market. However, with increased regulatory concerns in this market, it is of key importance to understand the competitive forces in the ad-tech market.

In this literature, some papers have studied the incentives of auctioneers to retain information in ad auctions. Rafieian and Yoganarasimhan (2021) study theoretically and empirically an ad network in the mobile apps market that can adopt various targeting regimes differing in their granularity. Decarolis et al. (2023) find that platforms running ad auctions have the incentive to obfuscate data to increase their revenues. Then, D'Annunzio and Russo (2023) model ad intermediaries managing advertisement allocation, endogenizing the outsourcing decision of publishers and the choice of an intermediary to disclose consumer information to advertisers in ad auctions. The present paper builds on D'Annunzio and Russo (2023) by considering competition between advertising intermediaries and the impact of integration between an advertising intermediary and a publisher on the adtech tax and investment in content.

We also add to the emerging literature on competition between intermediaries. Sharma et al. (2019) consider two competing ad intermediaries that are horizontally differentiated. In such a setting, they consider the impact of regulation on publishers. Decarolis et al. (2024) study search auctions and find that advertisers find it profitable to use platforms with more data and more sophisticated algorithms. We contribute to the literature by studying the impact of integration in the content market on publisher innovation and the ad tech tax.

Our work is also related to the extant literature on multi-homing and its affects on the advertising market (Ambrus et al. (2016), Athey et al. (2018), D'Annunzio and Russo (2020), Affeldt et al. (2021) and D'Annunzio and Russo (2023)). These papers build on the insight that consumers exposed to multiple ads on multiple outlet result in inefficiencies in ad campaigns. The closest work to ours is D'Annunzio and Russo (2023), which we build upon.

This paper studies the impact on competition and welfare of integration between an ad intermediary and a publisher. The impact of integration between a platform and its client is well studied (Mathewson and Winter (1987), Ordover et al. (1990), Hart et al. (1990), Bolton and Whinston (1991), Segal and Whinston (2000), Spector (2011) among others). The anticompetitive conduct can occur in two main ways. Either by foreclosing access to an important input to competing downstream rivals or when the vertically merged downstream firm sources exclusively from its own supplier and this shuts down competing input manufacturers. We contribute to the latter stream of literature.[7]

Finally, we also contribute to the growing policy relevant literature on regulation in the ad-tech sector. Witte and Kraemer (2023), Srinivasan (2020), Latham et al. (2021). Witte and Kraemer (2023) discuss multiple anti-competitive concerns associated with Google's dominance in the ad-tech market and propose regulatory remedies. Latham et al. (2021) considers the ad-tech market in stylized setting and discusses how anti-competitive conduct reinforce each other. Our paper builds on these ideas and formalizes them in a game theoretic model, providing clear mechanism for the claims presented in these influential works.

3 The model

We consider a setting with two intermediaries in the advertising market, denoted by $\{I_1, I_2\}$, and two digital publishers, denoted by $\{P_1, P_2\}$. In the following, we use superscript $i \in \{1, 2\}$ to refer to the intermediaries, and subscript $p \in \{1, 2\}$ to refer to the publishers. The publishers offer free content to consumers and sell impressions to advertisers via the intermediaries. We consider two scenarios: one where publishers and intermediaries are vertically separated (the VS scenario), and one where intermediary I_1 is vertically integrated with publisher P_1 (the VI scenario), while the other firms are separated. Figure 1 represents the two market structures.

[7]More recent paper on vertical integration, exclusive provision in markets featuring network effects include Weeds (2016), D'Annunzio (2017), Carroni et al. (2023) and Choi and Jeon (2023) among others.

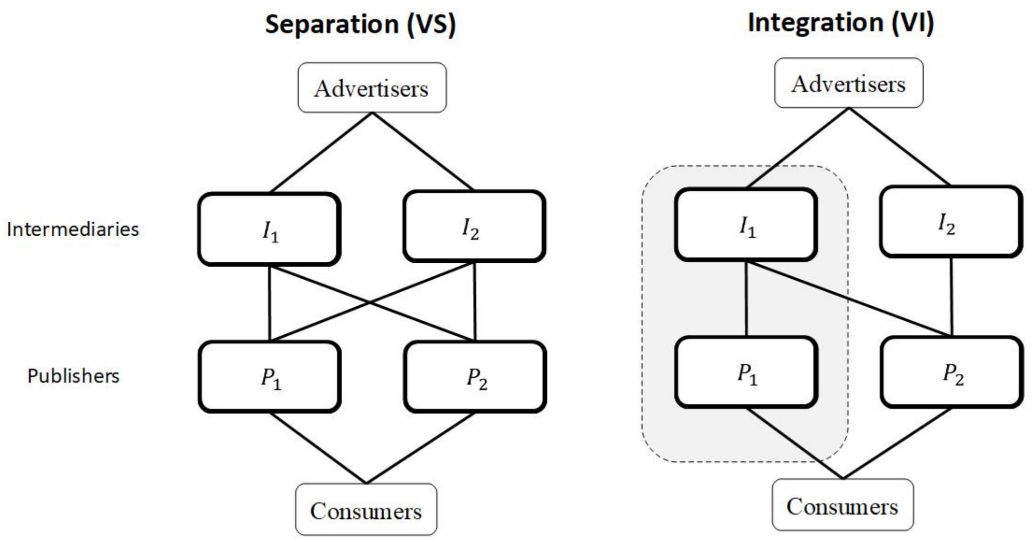

Figure 1: Market Structures

Consumers. There is a unit mass of consumers. Each obtains the following net utility from browsing the content provided by publisher p

$$V_p(u_p, q_p) = \underbrace{u_p}_{\text{Consumer type}} + \underbrace{\gamma q_p}_{\text{Quality}} - c \text{ for } p \in \{1, 2\},$$

where u_1 and u_2 are idiosyncratic consumer preferences for publisher P_1 and P_2, respectively. We assume that u_1 and u_2 are non-negative and distributed according to a joint distribution with smooth density $h(u_1, u_2)$. The variable q_p is the quality of content on publisher p, with $\gamma > 0$ being the consumer's marginal utility from content quality. Finally, c is the disutility consumers experience from being exposed to ads on that content (assumed identical across publishers without loss).

We denote the demand from consumers that visit only publisher p (single-homers) by D_p, the demand from consumers that visit both publishers (multi-homers) by D_{12}, and the mass of consumers visiting no publishers by D_0. These demands are specified as follows:[8]

$$D_p(q_p, q_{-p}) = Pr(V_p \geq 0, V_{-p} < 0) \text{ for } p, -p \in \{1, 2\} \text{ and } p \neq -p, \quad (1)$$

$$D_{12}(q_1, q_2) = Pr(V_1 \geq 0, V_2 \geq 0), \quad (2)$$

[8]In the following, we omit the arguments of the demand functions for ease of exposition, unless strictly necessary.

$$D_0(q_1, q_2) = Pr(V_1 < 0, V_2 < 0). \quad (3)$$

The following figure illustrates the values of u_1 and u_2 such that individuals visit either one, two, or no publishers.

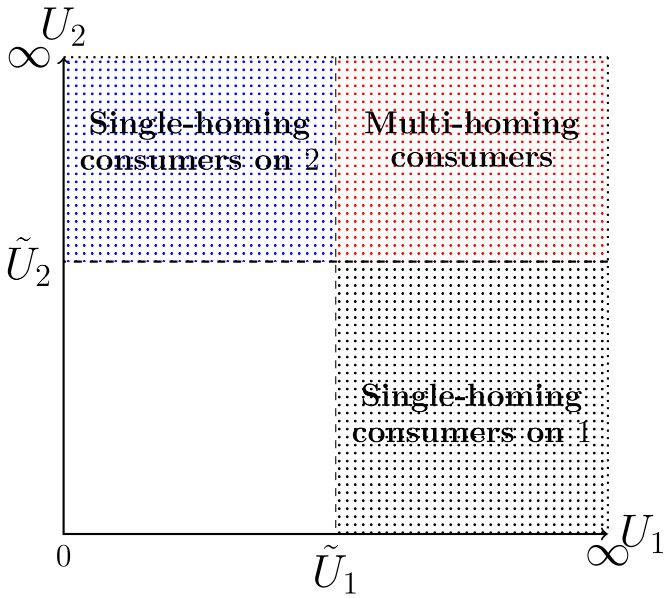

Figure 2: Single-homing and multi-homing consumers.

The area shaded with blue dots represents the values of u_1 and u_2 such that consumers visit only publisher P_2 (single-homing). Similarly, the area shaded with black dots represents the values of u_1 and u_2 such that consumers visit only publisher P_1. The area shaded with red dots represents the values of u_1 and u_2 such that consumers multi-home.

This demand system has the following properties. When q_p increases, some consumers who were not browsing any content and some that were single-homing on the other publisher $-p$ start to visit publisher p, so that $\frac{\partial D_p}{\partial q_p} \geq 0$, $\frac{\partial D_{12}}{\partial q_p} \geq 0$ and $\frac{\partial D_{-p}}{\partial q_p} \leq 0$. Note that the composition of publisher's $-p$ audience changes with q_p, but the total size of its audience remains the same, because $\frac{\partial D_{12}+D_{-p}}{\partial q_p} = 0$.[9] This discussion is demonstrated in the following Figure 3 which shows the effect of an increase in q_1 on consumer types.

[9] The model also allows the possibility that all consumers visit at least one publisher, so that $D_0 = 0$.

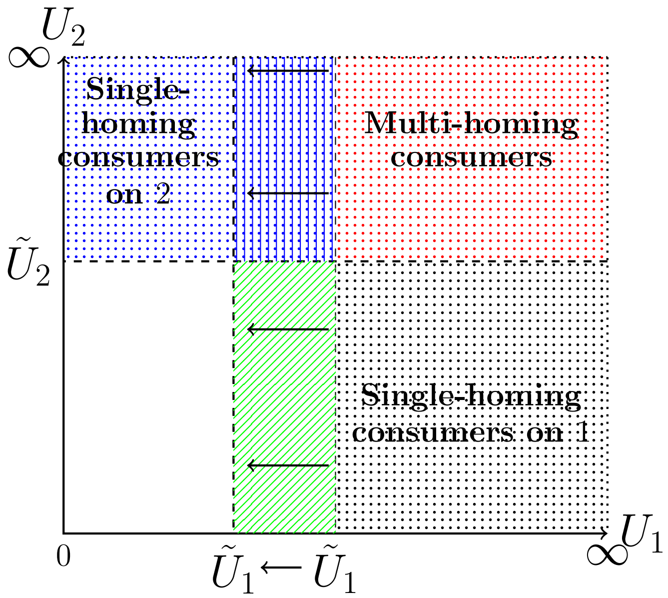

Figure 3: The impact of an increase in q_1 on consumer types.

Advertisers. There are $n > 2$ advertisers that want to inform consumers about their products.[10] All advertisers obtain the same return, v, from informing a consumer. This is private information of the advertiser. We assume that one ad impression on a consumer is enough to inform her/him, so hitting the same consumer with the same ad more than once is wasteful. This assumption captures the diminishing returns from hitting a consumer with the same ad (Ambrus et al., 2016; Athey et al., 2018; D'Annunzio and Russo, 2020, 2023).[11] In Appendix C we propose an alternative version of the model where we relax the assumption of diminishing returns to ads and consider ad targeting based on consumer product preferences, showing that the same results hold.

Publishers. The publishers choose the quality of their content, q_p, which entails an increasing and convex cost $k_p(q_p)$. Each consumer's visit generates an impression, so a single-homer (respectively, a multi-homer) is exposed to one impression (respectively, two impressions, one on each publisher). To focus on the interaction between the adtech industry and the provision of digital content, we assume the publishers cannot sell their ads directly to advertisers. For simplicity, we assume the publishers do not know the value of v, nor its distribution. This assumption rules out the possibility that the publishers

[10] In this setting, with two or fewer advertisers, the intermediaries may prefer not to disclose information about multi-homing consumers to avoid market-thinning effects (D'Annunzio and Russo, 2023). We assume that the number of advertisers is larger to bypass these concerns.

[11] We assume the value of the second impression is zero, but what matters for our results is that a repeated impression has less value to an advertiser than the first one. This assumption captures that advertisers value frequency management. For anecdotal evidence on this issue, see `https://tinyurl.com/ycyfphk4` and the concerns expressed by industry bodies (`https://tinyurl.com/48wec992`) and ad agencies (`https://tinyurl.com/bdh4hc5a`).

impose a reservation price for their impressions, which is consistent with the publishers lacking visibility regarding the advertisers' actual bids for their impressions, and having far less information about consumers than intermediaries can collect. We return to this point Section 4.2.

Intermediaries. The intermediaries allocate ad impressions to the advertisers via first-price auctions.[12] When auctioning impressions, each intermediary shares with the advertisers the information it has on consumers. More specifically, if the intermediary collects bid requests from both publishers, it knows whether the consumer is a single- or a multi-homer. Moreover, if it controls the sale of both impressions on the same consumers, it shares with the advertisers whether they are buying a first or a second impression on the consumer. These pieces of information allow the advertisers to assess the valuation for the consumer, controlling the frequency of impressions on consumers when running cross-publisher advertising campaigns. However, their availability depends on the market configuration.

Under vertical separation, whenever an impression opportunity is generated on publisher p, the latter sends bid requests to both intermediaries. Under vertical integration, only P_2 sends bid requests to both intermediaries, while all impressions on the integrated publisher P_1 are auctioned exclusively by I_1. We assume that there is "self preferencing".[13]

When receiving a bid request for an impression from a publisher, each intermediary collects bids from the advertisers, and then sends its own bid to the concerned publisher. The publisher assigns the impression to the intermediary submitting the highest bid, and the latter in turn allocates the impression to the advertiser that submitted the highest bid. We assume the publishers and intermediaries allocate the impressions randomly among the top bidders if their bids are equal. We denote by b_p^i an advertiser's bid for an impression on publisher p when auctioned at intermediary i, and by B_p^i the bid that intermediary i sends to publisher p for that impression. Note that, for any impression, an intermediary's bid may not coincide with the highest bid the intermediary receives from the advertiser. That is, each intermediary can *shade* its bid for an impression below what the winning advertiser actually submitted. This is consistent with current practices and linked to the lack of transparency on the adtech fees charged to digital publishers and advertisers by intermediaries (CMA, 2020).

In the vertical separation scenario, each intermediary can observe whether an impression falls on a single- or multi-homing consumer. This is because, as long as it receives bid requests from both publishers, the intermediary can track consumers across outlets

[12]Most intermediaries in the display advertising market run first-price auctions. For example, Google's ad exchange adopted this format in 2019.

[13]This assumption captures the current market configuration regarding "own and operated" advertising publishers of large ecosystems like Google. We relax the assumption in Section 6.

(e.g., with third-party cookies). Instead, in the VI scenario, the data generated by consumers visiting P_1 is only available to the integrated intermediary, I_1 (i.e., the data is not accessible outside the integrated firm's ecosystem). We also assume that an intermediary cannot observe which ad a consumer is exposed to if the impression is sold by the other intermediary. This assumption captures the difficulty of managing the frequency of exposure to ads for multi-outlet advertising campaigns when using multiple intermediary platforms.

Timing. The timing of the game is as follows.

t=1 Publishers decide on quality levels q_p for $p \in \{1, 2\}$.

t=2 Consumers visit the publishers and all impression opportunities are generated simultaneously. Publishers send bid requests. In the VS regime, for each impression, the respective publisher sends a bid request to both intermediaries. In the VI regime, only P_2 does that.

t=3 Each intermediary collects bids b_p^i from the advertisers for each impression opportunity.

t=4 Intermediaries send bids to publishers. In the VS regime, the intermediaries submit a bid B_p^i to the publisher for every impression. The winning intermediary then sells the impression to the advertiser that placed the highest bid on its auction. In the VI regime, only P_2 receives bids B_2^i from both intermediaries.

t=5 All payments for impressions are made and consumers get exposed to ads.

The equilibrium concept we adopt is Subgame-Perfect Nash Equilibrium. We make two assumptions to avoid multiplicity at stage 3, where advertisers decide their bidding strategies. First, we assume that, when indifferent between multiple strategies that yield identical profit, advertisers prefer *one-stop shop* campaigns, i.e., to buy impressions from a single intermediary.[14] Hence, we restrict attention to equilibria such that no advertiser can deviate unilaterally by acquiring all its impressions from the same intermediary while earning the same profit.[15] Among the equilibria that survive the above refinement, we restrict attention to those such that no advertiser can deviate by acquiring a larger volume of impressions while making the same profit.

[14] The market study by CMA (2020) reports in paragraph 5.219 that using a single demand side platform (DSP) for a given campaign allows the advertiser to manage frequency caps over the entire campaign and facilitates audience management and reporting. As a result, most large advertisers tend to use a single DSP, which is consistent with the equilibrium outcomes of our model.

[15] This refinement is consistent with anecdotal evidence about the difficulties that advertisers face when running campaigns using multiple platforms (see footnote 14).

4 Analysis

We begin from the case where intermediaries and publishers are vertically separated (VS scenario). Next, we consider the case where I_1 and P_1 are integrated (VI scenario). Finally, we compare the market outcomes and welfare in the two scenarios. Note that in either scenario, given the quality levels q_p, consumer demand and homing behavior in Stage 2 are as characterised in equations (1) and (2) and thus omitted.

4.1 Vertical separation (VS)

Before proceeding with the analysis, recall that in this scenario both intermediaries receive bid requests from each publisher for every impression (left panel of Figure 1).

4.1.1 Stage 4

At this stage, each intermediary submits a bid B_p^i for each impression available on publisher p. Let $\overline{b}_p^i$ be the highest bid received by intermediary i for the impression at stage 3, which is also the intermediary's willingness to pay to acquire and distribute the impression at stage 4. In a first-price auction, the impression is won by the intermediary with the highest willingness-to-pay and the equilibrium price, $\overline{B}_p$, equals the second-highest value of $\overline{b}_p^i$ for $i = 1, 2$.[16]

Lemma 1 *In the VS scenario, each impression on publisher P_p (for $p \in \{1, 2\}$) is acquired by the intermediary that receives the highest bid from the advertisers, i.e.,* $\max(\overline{b}_p^2, \overline{b}_p^1)$*. The equilibrium price of the impression paid to the publisher, $\overline{B}_p$, equals* $\min(\overline{b}_p^2, \overline{b}_p^1)$*. The intermediary acquiring and distributing the impression thus earns* $\max(\overline{b}_p^2, \overline{b}_p^1) - \min(\overline{b}_p^2, \overline{b}_p^1)$*.*

4.1.2 Stage 3

Consider an advertiser's willingness to pay for an impression on publisher p sold by intermediary i. Given single-homers are exposed to only one ad, there is no risk of repetition, implying that the advertiser is willing to pay v for any impression on these consumers. However, if the impression falls on a multi-homer, this consumer may already be exposed to the same ad on the other publisher. From the perspective of the advertiser, this is wasteful. In particular, if the advertiser acquires some impressions that hit multi-homers on the other publisher and these impressions are distributed by the other intermediary, there is a positive probability that the consumer is exposed to the same ad twice. This is because each intermediary does not observe which ads (if any) the consumer is exposed to if those are distributed by the other intermediary. Therefore, the advertiser's willingness

[16] The outcome is identical to that of a second-price auction, in accordance with the revenue equivalence principle (Myerson, 1981).

to pay for impressions on publisher p sold by intermediary i that fall on a multi-homer is $v(1-\delta_{-p}^{-i}(MH))$, where $\delta_{-p}^{-i}(MH)$ is the probability that the consumer is already exposed to the ad on the other publisher and is equal to the share of impressions on multi-homers that take place on the other publisher, $-p$, that are acquired by the advertiser via the other intermediary, $-i$.

The above discussion indicates that, due to the risk of repetition, impressions on multi-homers sold by different intermediaries on different publishers are perceived as substitutes by the advertisers. Given this substitutability, we find that in equilibrium each advertiser uses only one intermediary (see Appendix A.1). Intuitively, in this way advertisers can avoid repetition across outlets and maximise the efficiency of their campaigns.[17] In this situation, each intermediary has sufficient information to avoid repeating impressions on the same consumer. That is, by using a single intermediary for its campaign, each advertiser ensures that the intermediary has full control over the frequency of exposure to its ads on multiple publishers.[18]

The equilibrium price of each impression, whether on single- or multi-homing consumers, is v. To grasp the intuition, consider that information about whether an impression hits a consumer who has already been exposed to the same message when visiting another publisher either boosts an advertiser's willingness to pay to v (if the impression is on a consumer who has not seen it yet), or depresses it to zero (if the impression is repeated). However, because there are more than two advertisers and at most two impressions per consumer, providing this information enables the intermediary to have a bid equal to v for each impression. In other words, when an advertiser uses a single intermediary, it has complete certainty about whether it is already acquiring an impression on a given consumer. Given an equilibrium price of v for each impression, no advertiser can unilaterally deviate and obtain a higher profit.[19]

Summing up, in the VS scenario, each advertiser avoids the risk of repetition by using a single intermediary for its ad campaigns, giving it full control on the frequency of exposure of consumers to its impressions. The following lemma summarizes these findings:

Lemma 2 *In the VS scenario, each advertiser buys all its impressions from one inter-*

[17]If $n \geq 4$, another equilibrium candidate exists: each advertiser uses multiple intermediaries but a single publisher for impressions that hit multi-homers, though not all advertisers use the same one. This equilibrium entails the same market outcomes in terms of ad prices and profits for advertisers, publishers and intermediaries, and are thus equivalent for the purpose of our analysis. However, we exclude this equilibrium because of the one-stop shopping refinement we use.

[18]There is no equilibrium where all advertisers use the same intermediary. As we show Appendix A.1, at least one advertiser could deviate bidding to a different intermediary and get more impressions for the same profit. Given equal bids from the intermediaries, the publishers would allocate half their impressions to each intermediary and each advertiser would get an equal share of the impressions.

[19]The advertisers could buy impressions on single-homing consumers from different intermediaries and use just one intermediary for impressions on multi-homers. Our assumption that advertisers prefer *one-stop shop* campaigns rules these equilibria out.

mediary and the equilibrium price of all impressions is $\overline{b}_p^i = v$*, for any* i *and* p*.*

Combining the above information with Lemma 1, we conclude that each intermediary pays a price equal to $\overline{B}_p(VS) = v$ for every impression to each publisher. Hence, in this scenario neither the advertisers nor the intermediaries make any profit. All the surplus from advertising is captured by the publishers. This implies that there is no adtech tax denoted in this scenario — i.e., $(v - \overline{B}_p(VS) = 0)$.

Lemma 3 *In the* VS *scenario, each intermediary pays a price* $\overline{B}_p(VS) = v$ *for every impression to each publisher and there is zero adtech tax. The publishers obtain the full revenue from their impressions. Intermediaries and advertisers make zero profits.*

The intuition is that in the VS scenario the intermediaries are on a "level playing field" from the perspective of the publishers and advertisers. As a consequence, when bidding to acquire the right to distribute the impressions, the intermediaries compete away all the revenue they extract from the advertisers.

4.1.3 Stage 1

At the first stage of the game, publisher p chooses its quality level maximizing its net profit:

$$\pi_p(VS) = \overline{B}_p(VS)(D_p + D_{12}) - k_p(q_p) = v(D_p + D_{12}) - k_p(q_p), \quad p = 1, 2. \tag{4}$$

Differentiating the above with respect to q_p yields the following first order conditions.

$$\frac{\partial \pi_p(VS)}{\partial q_p} = v\left(\frac{\partial D_p}{\partial q_p} + \frac{\partial D_{12}}{\partial q_p}\right) - \frac{\partial k_p(q_p)}{\partial q_p} = 0, \quad p = 1, 2. \tag{5}$$

Note that, by the properties of the total demand presented above (see Figure 3), the terms in each first order condition do not depend on the quality set by the other publisher, q_{-p}. Solving the above system of first order conditions simultaneously gives us the equilibrium quality levels in the VS scenario, that we denote by $q_p(VS)$.

4.2 Vertical integration (VI)

Suppose now that I_1 and P_1 are vertically integrated (right panel of Figure 1). In this scenario, only intermediary I_1 sells the impressions generated by P_1 and can access consumer data there. Hence, only I_1 can track consumers across publishers.

4.2.1 Stage 4

The analysis of this stage is very similar to that of Stage 4 in the VS scenario, except that only publisher P_2 makes its ad inventory available to both intermediaries. We can therefore state the following:

Lemma 4 *In the VI scenario, any impression generated on publisher P_2 is acquired by the intermediary that collects the highest bid from the advertisers, i.e., $max(\bar{b}_2^2, \bar{b}_2^1)$. The equilibrium price paid by the intermediary to publisher P_2 equals $\min(\bar{b}_2^2, \bar{b}_2^1)$. Hence, the intermediary acquiring the impression earns a margin equal to $\max(\bar{b}_2^2, \bar{b}_2^1) - \min(\bar{b}_2^2, \bar{b}_2^1)$.*

4.2.2 Stage 3

We first describe the advertisers' willingness to pay for impressions on each publisher, conditional on the intermediary that makes them available. Next, we describe the equilibrium bidding strategies of the advertisers.

Advertisers' WTP. Under vertical integration, an advertiser's willingness to pay for an impression auctioned by intermediary I_2 which can only be on publisher P_2 due to vertical integration between I_1 and P_1 is given by:

$$w_2^2 = \underbrace{v\frac{D_2}{D_2 + D_{12}}}_{\substack{\text{Expected value of} \\ \text{impression on single-homer}}} + \underbrace{v\left(1 - \delta_1^1(MH)\right)\frac{D_{12}}{D_2 + D_{12}}}_{\substack{\text{Expected value of} \\ \text{impression on multi-homer}}} = v\left(1 - \frac{D_{12}}{D_2 + D_{12}}\delta_1^1(MH)\right). \tag{6}$$

To understand this expression, note that the advertiser cannot condition its valuation on whether the consumer is a single- or a multi-homer, as this information is unavailable to I_2. If the consumer is a single-homer, the impression is worth v. Instead, a multi-homer may already be exposed to the same message when visiting P_1. Given impressions on P_1 are only distributed by I_1, the probability of repetition equals the share of impressions hitting multi-homers on P_1 that the advertiser acquires via I_1, which we denote by $\delta_1^1(MH)$. Observe that, due to the risk of repetition, an increase in the share of multi-homers in the audience of publisher P_2 lowers the advertisers' willingness to pay for the impressions auctioned by I_2, all else given. In sum, the advertisers' willingness to pay is the expected value of hitting a consumer that has not been advertised by intermediary I_1 — i.e., $v\left(1 - \frac{D_{12}}{D_2+D_{12}}\delta_1^1(MH)\right)$ where $\left(1 - \frac{D_{12}}{D_2+D_{12}}\delta_1^1(MH)\right)$ is probability of not hitting a multi-homer on P_2 that has already been advertised to via I_1.

Consider now the willingness-to-pay for an impression on publisher P_2 when auctioned by I_1. Unlike I_2, this intermediary enables the advertisers to distinguish between single- and multi-homers. In addition, I_1 can inform the advertiser about whether it is buying an impression on the same multi-homer when that consumer also visits P_1, because I_1 has full control over those impressions. Hence, the advertiser's willingness to pay for an impression auctioned by I_1 on P_2 is $w_2^1(SH) = v$ if the consumer is a single-homer, and $w_2^1(MH) = v$ if the consumer is a multi-homer and the advertiser is not buying another impression hitting the same consumer on P_1.

As in the VS scenario, being able to provide information about which ads a consumer sees on the other publisher allows the intermediary to maximise the bids obtained for the impression. However, in the VI scenario only I_1 has this ability. Thus, leveraging the exclusive data on consumers that visit P_1 enables I_1 to generate higher bids than I_2 when auctioning the impressions from P_2, even though P_2 and I_1 are not integrated.

Let us now turn to the willingness-to-pay for impressions auctioned by I_1 on publisher P_1. The willingness-to-pay for an impression on a single-homer is $w_1^1(SH) = v$. If the consumer is a multi-homer, I_1 can observe whether it is delivering to the same consumer the same impression on publisher P_2 only if it is not distributed by I_2. Consequently, we have $w_1^1(MH) = v\,(1 - \delta^2)$, where δ^2 is the share of impressions acquired by the advertiser on P_2 via I_2. Therefore, if the advertiser buys some impressions via I_2, intermediary I_1 cannot fully optimize the frequency of its impressions on P_1 despite having exclusive access to consumer data on that publisher.

Equilibrium bidding strategies. The above discussion indicates that, like in the VS scenario, impressions that hit multi-homers on different publishers auctioned by different intermediaries are perceived as substitutes by the advertisers.

Intuitively, the advertisers can avoid the risk of repetition on multi-homers when running an ad campaign on multiple outlets acquiring impressions only via intermediary I_1, giving it full control over the frequency of their impressions. In this equilibrium candidate, the advertisers pay v for each impression they acquire. Given $n > 2$ advertisers and no more than two impressions per consumer, for each impression there are at least two advertisers willing to pay v. Consequently, each advertiser makes zero profit, but cannot unilaterally deviate to a more profitable bidding strategy.[20]

The bidding strategies in equilibrium are as follows. Each advertiser buys impressions only from intermediary I_1, bidding v for any impression on single-homers. Moreover, each advertiser bids v for any impression on multi-homers sold by I_1, unless it is already acquiring an impression on the same consumer taking place on the other publisher, in which case the advertiser bids zero. Therefore, the highest bid collected by I_1 for every impression equals v, i.e., $\overline{b}_p^1(SH) = \overline{b}_p^1(MH) = v$, for any p. Finally, the highest bid for any impression collected by I_2, $\overline{b}_2^2$ equals w_2^2, as in (6), with $\delta_1^1(MH) = 1/n$.[21]

[20]The advertisers may avoid the risk of repetition on multi-homers when running an ad campaign on multiple outlets by also acquiring from I_1 only the impressions that hit single-homers on P_1. This equilibrium candidates yield the same ad price and advertisers' profits. However, we rule out this equilibrium candidate where advertisers multi-home on the two intermediaries by the assumption that the advertisers prefer "one stop shopping campaigns" when indifferent. Hence, in the following we focus on the equilibrium where they all place winning bids only on I_1.

[21]One may wonder whether the advertisers find it profitable to deviate by offering a bid lower than v on I_2. However, we can show that no advertiser will find such deviations profitable. This is because if they did so $\overline{b}_2^2$ would not be high enough to ensure that I_2 actually distributes those impressions.

Lemma 5 *In the VI scenario, all advertisers buy impressions from I_1 only and the highest bid collected by this intermediary is $\bar{b}_p^1(VI) = v$, for $p = 1, 2$. Intermediary I_2 collects the following highest bid for any impression available*

$$\bar{b}_2^2(VI) = v\frac{D_2}{D_2 + D_{12}} + v\frac{D_{12}}{D_2 + D_{12}}\left(1 - \frac{1}{n}\right) = v\left(1 - \frac{D_{12}}{D_2 + D_{12}}\frac{1}{n}\right).$$

Combining this information with Lemma 4, we see that any impression generated on publisher P_2 is acquired and distributed by intermediary I_1 at a price $\overline{B}_2(VI) = \bar{b}_2^2(VI) = v\left(1 - \frac{D_{12}}{D_2+D_{12}}\frac{1}{n}\right)$.

Lemma 6 *In the VI scenario, the equilibrium payment by this intermediary I_1 to publisher P_2 for each impression is $\overline{B}_2(VI) = \bar{b}_2^2(VI) = v\left(1 - \frac{D_{12}}{D_2+D_{12}}\frac{1}{n}\right)$.*

Observe that the equilibrium payment per impression by I_1 to P_2 is lower under VI than under VS — i.e., $\overline{B}_2(VI) < v$. Due to its data advantage over the other intermediary, I_1 is able to "shade" its bids for any impression to publisher P_2, i.e., offer less than the bids it receives for the same impression from the advertisers. Thus, I_1 earns a margin on each impression on publisher P_2 equal to $\bar{b}_2^1(VI) - \bar{b}_2^2(VI) = v\left(\frac{D_{12}}{D_2+D_{12}}\frac{1}{n}\right) > 0$. In other words, the advantageous position of the integrated firm enables it to impose an *adtech tax*, which would not be feasible in the VS scenario. In our setting, this tax falls entirely on the independent publishers.

Notice also that the payment $\overline{B}_2(VI)$ to publisher P_2 decreases as the share of multi-homers within the audience of publisher P_2 increases. The reason is that more multi-homers reduce the advertisers' willingness to pay for impressions sold by the independent intermediary, I_2, due to the perceived risk of repetition. Thus, I_1 can squeeze more margin from each impression it distributes on publisher P_2. That is, the size of the adtech tax (per impression) increases with the share of multi-homers in the independent publisher's audience. Intuitively, if there were no multi-homers in P_2's audience, the inability of I_2 to track consumers on P_1 would not matter for the advertisers, as there would be no chance of repetition.

Proposition 1 (Integration and adtech tax.) *Unlike with vertical separation, there is a positive adtech tax under vertical integration, equal to $v - \overline{B}_2(VI) = v\frac{D_{12}}{D_2+D_{12}}\frac{1}{n}$, which increases in the share of multi-homing consumers in the audience of the third-party publisher.*

Before proceeding with the analysis, it is useful to briefly consider relaxing the assumption that P_2 cannot impose a reservation price for its impressions. To set such price, the publisher would need some knowledge about the value that advertisers place on reaching consumers, v. We assumed that this information is not available to the publishers, to capture the fact that they typically have no visibility on the outcome of the auctions for impressions run by the intermediaries, and lack sufficient data to estimate the advertisers' value for each individual consumer because they are unable to track consumers across websites. However, the publishers may have aggregate information about the distribution of v, which they can use to set a reservation price. We allow for this possibility in Appendix A.4. We augment the baseline model by assuming that, at a preliminary stage, v is drawn from a distribution $F(v)$ which is common knowledge. Intuitively, by imposing a reservation price, P_2 can recover some of the adtech tax captured by I_1. However, P_2 cannot increase the reservation price too much, because it runs the risk that the advertisers find it too expensive to place ads on its website. Therefore, given P_2 cannot observe the realization of v, the qualitatively results in Proposition 1 would not change.

4.2.3 Stage 1

Let us now consider the choice of quality levels. Each publisher chooses its quality provision to maximize its profit. The profit of the vertically integrated firm and the publisher P_2 are respectively as follows

$$\begin{aligned}\pi_1(VI) &= v(D_1 + D_{12}) + \overbrace{(v - \overline{B}_2(VI))}^{\text{Ad-tech tax}}(D_2 + D_{12}) - k_1(q_1), \\ &= v(D_1 + D_{12}) + v\frac{D_{12}}{n} - k_1(q_1), \end{aligned} \tag{7}$$

$$\pi_2(VI) = \overline{B}_2(VI)(D_2 + D_{12}) - k_2(q_2) = v(D_2 + D_{12}) - v\frac{D_{12}}{n} - k_2(q_2). \tag{8}$$

Differentiating the profit of Publisher P_1 as presented in equation (7) with respect to q_1, and recalling that $\frac{\partial D_2}{\partial q_2} + \frac{\partial D_{12}}{\partial q_1} = 0$, yields

$$\frac{\partial \pi_1(VI)}{\partial q_1} = v\left(\frac{\partial D_1}{\partial q_1} + \frac{\partial D_{12}}{\partial q_1}\right) - \underbrace{\frac{\partial \overline{B}_2(VI)}{\partial q_1}(D_2 + D_{12})}_{\substack{\text{Bid shading} \\ \text{effect } (-)}} - \frac{\partial k_1(q_1)}{\partial q_1} = 0. \tag{9}$$

The key difference between this derivative and the corresponding first-order condition in the VS scenario in (5) is the "bid shading effect", capturing the fact that higher quality investment increases the share of multi-homers in P_2's audience. This increases I_1's margins for impressions at P_2 and increases profitability. The ability to shade bids more aggressively encourages further investments in quality. We have

$$\frac{\partial \overline{B}_2(VI)}{\partial q_1} = \frac{-v}{n(D_2 + D_{12})^2}\frac{\partial D_{12}}{\partial q_1} \leq 0.$$

Consider now the quality investment choice of the independent publisher. Differentiating its profit with respect to q_2 yields

$$\frac{\partial \pi_2(VI)}{\partial q_2} = v\left(\frac{\partial D_2}{\partial q_2} + \frac{\partial D_{12}}{\partial q_2}\right) - \frac{v}{n}\frac{\partial D_{12}}{\partial q_2} - \frac{\partial k_2(q_2)}{\partial q_2} = 0. \tag{10}$$

The independent publisher's first-order conditions contain an extra negative term when compared to (5) in the VS scenario. The reason is again related to the adtech tax. Investing in content quality increases the size of the independent publisher's audience, but also the share of multi-homers within it. As we have seen, this latter effect inflates the adtech tax, and thus reduces the ability of the publisher to monetize its impressions. Thus, the incentives to invest in quality can be lower.

Solving simultaneously the system of FOCs in (9) and (10) yields solution which we denote by the vector of equilibrium quality levels, $\mathbf{q(VI)} \equiv (q_1(VI), q_2(VI))$.

4.3 Comparing quality investments in VS and VI

We are now in a position to study how vertical integration affects the way publishers invest in content quality. Comparing (9) to (5), we have seen that there is an additional term that positively affects the investment in quality of the vertically integrated firm. By contrast, the additional term in (10) compared to (5) is negative. Recalling that $\frac{\partial D_{12}}{\partial q_p} \geq 0$ and $\frac{\partial D_{12}+D_{-p}}{\partial q_p} = 0$ by the properties of the demand system, we can thus conclude the following

Proposition 2 (Quality levels.) *Under VI, the quality level chosen by P_1 is (weakly) higher than under VS, whereas the quality level chosen by P_2 is (weakly) lower — i.e., $q_1(VI) \geq q_1(VS)$ and $q_2(VI) \leq q_2(VS)$.*

As we have seen in Proposition 1, integration enables the integrated firm to capture part of the ad revenue generated by the independent publisher via the adtech tax. Therefore, P_2's incentive to invest in quality decreases with respect to the VS scenario. While higher q_2 expands the size of P_2's audience, it also increases the share of multi-homers within such audience, which determines the size of the adtech tax. Indeed, expression (10) would be identical to (5) if all consumers single-homed.

The effect of integration on investment by P_1 is less intuitive, because through the adtech tax the integrated firm internalizes the effect of its own investment on the ad revenue generated by the independent publisher. If this effect is negative, it may discourage P_1's quality investment. However, we demonstrate that the opposite occurs. The reason is that, while q_1 does not affect the total size of P_2's audience, it changes its composition by increasing the share of multi-homers. Hence, q_1 does not change the total ad revenue generated by P_2, but increases the part of this revenue appropriated by I_1 via the adtech tax.

Overall, Proposition 2 shows that the effect of integration between adtech intermediaries and digital publishers on quality investment is quite nuanced. This effect arises from the incentive of the integrated firm to increase the adtech tax by changing the composition of the independent publisher's audience.

5 Welfare analysis

We now analyze the effect of vertical integration on total surplus and on its distribution among consumers, platforms and advertisers. The following expressions describe consumer surplus for single-homers of P_1 and P_2, as well as multi-homers, respectively

$$CS_1(q_1, q_2) \triangleq \int_0^{c-\gamma q_2} \int_{c-\gamma q_1}^{\infty} (u_1 + \gamma q_1 - c) h(u_1, u_2) du_1 du_2, \tag{11}$$

$$CS_2(q_2, q_1) \triangleq \int_0^{c-\gamma q_1} \int_{c-\gamma q_2}^{\infty} (u_2 + \gamma q_2 - c) h(u_1, u_2) du_1 du_2, \tag{12}$$

$$CS_{12}(q_1, q_2) \triangleq \int_{c-\gamma q_1}^{\infty} \int_{c-\gamma q_2}^{\infty} (u_2 + \gamma q_2 - c + u_1 + \gamma q_1 - c) h(u_1, u_2) du_1 du_2. \tag{13}$$

Thus, total consumer surplus is $CS(q_1, q_2) = CS_1 + CS_2 + CS_{12}$.

As for firm profits, observe that in both the VS and VI scenario total profit coincides with the surplus derived from advertising net of the cost of quality investment, as all payments between firms cancel out and firms sustain no other costs. We call it advertising surplus and denote it as AS. In both scenarios, the equilibrium we characterized is such that each impression generates a surplus of v for the advertisers.

Total welfare is the sum of firm profits (i.e., advertising surplus AS) and consumer surplus. Hence, we can write

$$W(q_1, q_2) = v(D_1(q_1, q_2) + D_2(q_2, q_1) + 2D_{12}(q_1, q_2)) - k_2(q_2) - k_1(q_1) + CS(q_1, q_2). \tag{14}$$

This surplus depends on content quality levels in the markets. For a given level of quality, expression (14) shows that welfare is unaffected by vertical integration and the ensuing adtech tax. Indeed, for given quality levels, consumer surplus does not change, and integration only redistributes advertising profits from the independent firms to the integrated one.

However, vertical integration does affect welfare through its effect on content quality. Let us first consider the change in the profits of firms. We know from Proposition 2 that integration pushes the quality investment levels of the two publishers in opposite directions. Despite the fact that the integrated firm invests more in quality and has thus higher costs than in the VS scenario, its total profit is higher than the sum of the profits of P_1 and I_1 when separate. By contrast, the profit of the independent publisher, P_2, is lower under VI than under VS. Indeed, we have

$$[v(D_2+D_{12})-k_2]\,|_{\mathbf{q(VS)}} > [v(D_2+D_{12})-k_2]\,|_{\mathbf{q(VI)}} > \left[v\left(D_2+D_{12}-v\frac{D_{12}}{n}\right)-k_2\right]|_{\mathbf{q(VI)}}. \tag{15}$$

Proposition 3 (Integration and the distribution of profits.) *The profit of vertically integrated firm is higher under VI than under VS. Instead, the profit of the independent publisher is lower under VI than under VS. The independent intermediary and advertisers make zero profit in both scenarios.*

Consider now the changes in consumer surplus. Given integration induces an increase in q_1, but a decrease in q_2, the net effect on consumer surplus is ambiguous. Therefore, we are unable to conclude in general whether integration increases total surplus. In Section 5.2 we study this issue taking into account the uniform distribution.

5.1 Comparing equilibrium and social optimum

The first best quality levels denoted by the vector $\mathbf{q}^* \equiv (q_1^*, q_2^*)$ that maximize (14), satisfy the following system of equations

$$\frac{\partial W}{\partial q_i} = v\left(\frac{\partial D_1}{\partial q_i}+\frac{\partial D_2}{\partial q_i}+2\frac{\partial D_{12}}{\partial q_i}\right)+\frac{\partial CS}{\partial q_i}-\frac{\partial k_i}{\partial q_i} = v\left(\frac{\partial D_i}{\partial q_i}+\frac{\partial D_{12}}{\partial q_i}\right)+\frac{\partial CS}{\partial q_i}-\frac{\partial k_i}{\partial q_i} = 0, i=1,2, \tag{16}$$

where the second equality follows from $\frac{\partial D_{12}+D_{-i}}{\partial q_i} = 0$. We now compare $\mathbf{q}^*$ to $\mathbf{q(VS)}$. Evaluating the system of first-order derivatives of (16) with the investment levels under vertical separation $\mathbf{q(VS)}$, we find that:

$$\left.\frac{\partial W}{\partial q_i}\right|_{\mathbf{q(VS)}} = \left.\frac{\partial CS}{\partial q_i}\right|_{\mathbf{q(VS)}} > 0. \tag{17}$$

This inequality indicates that quality is under-supplied with vertical separation. This is not surprising as the publishers do not internalize the effect of quality on consumer surplus.

Let us now compare the socially optimal quality levels to the equilibrium levels under vertical integration. Evaluating the system (16) with the quality levels under vertical integration denoted by $\mathbf{q(VI)}$, we have:

$$\left.\frac{\partial W}{\partial q_1}\right|_{\mathbf{q(VI)}} = \left.\left(-\frac{v}{n}\frac{\partial D_{12}}{\partial q_1}+\frac{\partial CS}{\partial q_1}\right)\right|_{\mathbf{q(VI)}} \lesseqgtr 0, \tag{18}$$

$$\left.\frac{\partial W}{\partial q_2}\right|_{\mathbf{q(VI)}} = \left.\left(\frac{v}{n}\frac{\partial D_{12}}{\partial q_2}+\frac{\partial CS}{\partial q_2}\right)\right|_{\mathbf{q(VI)}} > 0. \tag{19}$$

Given $q_2(VI) < q_2(VS)$, the under-provision problem of publisher P_2 worsens under vertical integration. On the other hand, given $q_1(VI) > q_1(VS)$ the quality provided by publisher P_1 may either exceed the optimal level under vertical integration or fall short of it, i.e. $q_1(VI) \lesseqgtr q_1^*(q_2(VI))$, where $q_1^*(q_2(VI))$ denotes the optimal quality $q_2 = q_2(VI)$.

Proposition 4 *Social welfare given vertical integration can be higher or lower than under vertical separation.*

Although it redistributes profits away from independent firms, vertical integration does not necessarily have a negative effect on welfare. This is because the increased incentives to invest in quality by P_1 can alleviate the under-provision established for the separation scenario. In fact, over-provision of quality by P_1 may theoretically occur under integration. In the following, we present three examples demonstrating how the welfare properties of the equilibrium depend on the distribution of consumer preferences.

5.2 Examples

In the following, we consider that consumer preferences u_i follow a uniform distribution. Further, we consider the three cases (i) when consumer preferences are independent, (ii) negatively correlated and (iii) positively correlated. On the cost side, we assume that the investment technology is convex and is denoted by $k_p(\cdot) = \frac{q_p^2}{2}, \forall p \in \{1, 2\}$. We provide an overview of the results, and relegate the detailed analysis in Appendix B.

A. Independent preferences. Suppose consumer preferences for the publishers are independent and distributed as follows: $u_1 \sim \mathcal{U}[0, 1]$ and $u_2 \sim \mathcal{U}[0, 1]$. The investment levels under vertical separation are $q_p(VS) = \gamma v$ for $p \in \{1,\ 2\}$. The investment levels under vertical integration are

$$q_1(VI) = \frac{\gamma v\,(n(n+1-c) + \gamma^2 v(n-1+c))}{n^2 + v^2\gamma^4}, q_2(VI) = \frac{\gamma v\,(n(n-1+c) - v\gamma^2(n+1-c))}{n^2 + v^2\gamma^4}.$$

Note that the increase in investment by the integrated publisher is always lower than the decrease in investment by independent publisher — i.e., $|q_2(VI) - q_2(VS)| - |q_1(VI) - q_1(VS)| > 0$. This reduction in total value creation under vertical integration in the market affects consumer surplus and welfare. In particular, we find that consumer surplus and total surplus fall under vertical integration.

$$CS(VS) - CS(VI) = \frac{v^2\gamma^4(1-c+v\gamma^2)^2}{(n^2 + v^2\gamma^4)} > 0,$$

and

$$W(VS) - W(VI) = \frac{v^2\gamma^2(1+\gamma^2)(1-c+v\gamma^2)^2}{(n^2 + v^2\gamma^4)} > 0.$$

To gain intuition, first consider consumer surplus. Under vertical integration, the quality on publisher 1 increases while the quality invested by publisher 2 falls. As a consequence, consumers on publisher 1 are better off while consumers on publisher 2 are worse-off. However, the gain in surplus at the integrated published is lower than the loss at the independent publisher because the rise in quality at the integrated publisher is lower than the reduction in quality at the independent publisher. This directly translates into a total consumer surplus loss as the gains in surplus from vertical integration (at the integrated intermediary) are dominated by the consumer surplus loss for consumers of the non-integrated publisher. Thus, under vertical integration, consumer surplus decreases.

Now, consider welfare, defined as the sum of consumer surplus and welfare (in equation 14). Net surplus generated on the market is higher under vertical separation because total demand increases (see first term in 14), hence the number of impressions available under vertical separation is higher than under vertical integration. As a consequence, not only consumer surplus but also advertiser surplus is higher under vertical separation. Even if the total cost from quality investment may increase under vertical separation, the increase of consumer and advertiser surplus more than compensates it.

B. Negatively correlated preferences. Suppose now the distribution of preferences is such that $u_1 \sim [0,1]$ and $u_2 = 1 - u_1$. This is fundamentally a "Hotelling" setup where publisher P_1 is located at coordinate 1 and publisher P_2 is located at coordinate 0.[22] All consumers have a unit valuation for content on each publisher and the "transportation" cost (per unit of distance) is equal to one. The following figure depicts the consumer types who multi-home and single-home.

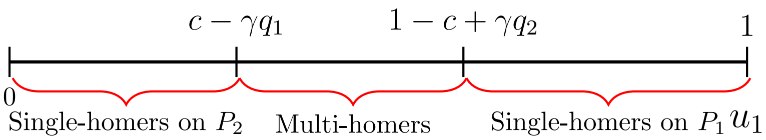

Figure 4: Single and multi-homing consumers.

The investment levels under vertical separation and under vertical integration are

$$q_p(VS) = \gamma v \text{ for } p \in \{1,\ 2\},$$

$$q_1(VI) = \frac{\gamma v(n+1)}{n},\ q_2(VI) = \frac{\gamma v(n-1)}{n}.$$

Note that the increase in investment by the integrated publisher is exactly equal to the decrease in investment by independent publisher — i.e., $|q_2(VI) - q_2(VS)| - |q_1(VI) - q_1(VS)| = 0$. This is a classical Hotelling result due to a one-to-one transfer. Comparing

[22]See Jullien et al. (2023) for more details.

consumer surplus and welfare, we find that consumer surplus increases with vertical integration, but total surplus can increase or decrease.

$$CS(VS) - CS(VI) = -\frac{v^2\gamma^4}{n^2} < 0, \quad \text{and } W(VS) - W(VI) = \frac{v^2\gamma^2(1-\gamma^2)}{n^2}.$$

Consider consumer surplus. In the two regimes, the mass of multi-homers and the surplus they get does not change, because the total quality from publishers stays the same. However, the mass of single-homers and their surplus changes. Users who single-home on publisher P_2 shrinks and are worse-off as the publisher invests less under vertical integration. Symmetrically, users who single-home on the integrated publisher increase and are better-off. In sum, this leads to an increase in consumer surplus after vertical integration.

The result on welfare is a bit nuanced. Recall, that welfare is the sum of consumer surplus and advertising surplus. Firstly, we find that the advertising surplus is higher under vertical separation (vis-á-vis). To be more concrete,

$$\Delta AS = \sum_{p=1,2}\left(D_p(VS) + D_{12}(VS) - D_p(VI) - D_{12}(VI) - \left(\frac{q_p(VS)^2}{2} - \frac{q_p(VI)^2}{2}\right)\right)$$

$$\Delta AS = \sum_{p=1,2}\left(-\left(\frac{q_p(VS)^2}{2} - \frac{q_p(VI)^2}{2}\right)\right) > 0. \tag{20}$$

Vertical integration does not increase the total number of impressions available but increases cost. This because only the investment of each individual publisher changes while the total investment level across them remains unchanged. As the investment costs are convex, an increase in investment by the integrated publisher results in increased investment cost. This increased investment cost due to reallocation of investment efforts negatively affects advertising surplus under vertical integration. However, the negative impact of advertising surplus on total welfare under vertical integration can be countervailed by its impact on consumer surplus. Specifically, when consumers' sensitivity to quality is high (when $\gamma > 1$), we find that total welfare can be higher under vertical integration than under vertical separation as the consumer surplus gains are greater than welfare losses due to increased investment costs. Else when $\gamma \leq 1$, total welfare is lower under vertical integration.

C. Positively correlated preferences. Now, suppose that $u_2 = \alpha u_1$ with $u_1 \sim [0,1]$ and $u_2 \sim [0,\alpha]$. All consumers such that $V_p \geq 0$ visit publisher p. We denote as $\bar{u}_1$ (resp. $\bar{u}_2$) the indifferent consumer on publisher 1 (resp. 2). Then, we can identify two cases, with a different demand structure: (i) $\bar{u}_1 < \bar{u}_2$, then $D_2 = 0$; (ii) $\bar{u}_1 > \bar{u}_2$, then $D_1 = 0$;
Case (i) $0 < \bar{u}_1 \leq \bar{u}_2$. Under this specification, the following figure depicts user types that single-home and multi-home.

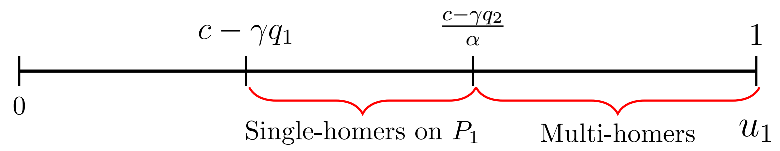

Figure 5: Single and multi-homing consumers under case (i) $\bar{u}_1 \leq \bar{u}_2$.

The investment levels under vertical separation and under vertical integration are

$$q_1(VS) = \gamma v,\ q_2(VS) = \frac{\gamma v}{\alpha},$$

$$q_1(VI) = v\gamma,\ q_2(VI) = \frac{\gamma v(n-1)}{n\alpha}.$$

Note that the investment level of publisher P_1 does not change in the two regimes. This implies that the utility of consumers who were single-homing on publisher P_1 in the two regimes does not change. Instead, the investment level of publisher P_2 falls under vertical integration. A direct implication is that multi-homing consumers are worse-off under vertical integration. Further, some of the consumers who were multi-homing under vertical separation switch behavior and start single-homing on P_1. This is directly due to a loss in utility from multi-homing under vertical integration. This impacts negatively consumer surplus and welfare. Specifically, we find that vertical integration reduces both consumer and total surplus, i.e.,

$$CS(VS) - CS(VI) = \frac{\gamma^2 v \left(v\gamma^2(2n-1) - 2\alpha n(c-\alpha)\right)}{2\alpha^3 n^2} > 0.$$

$$W(VS) - W(VI) = \frac{\gamma^2 v \left(v\gamma^2(2n-1) + v\alpha - 2n\alpha(c-\alpha)\right)}{2\alpha^3 n^2} > 0.$$

Case (ii) $\bar{u}_1 > \bar{u}_2$: Under this specification, the following figure depicts user types that single-home and multi-home.

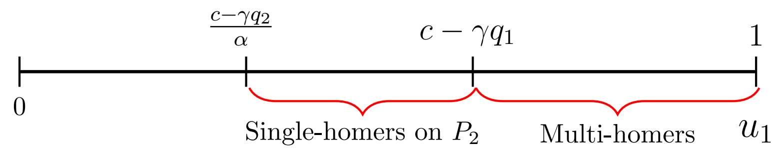

Figure 6: Single and multi-homing consumers under case (ii) $\bar{u}_1 > \bar{u}_2$.

The investment levels under vertical separation and under vertical integration are

$$q_1(VS) = \gamma v,\ q_2(VS) = \frac{\gamma v}{\alpha}$$

$$q_1(VI) = \frac{\gamma v(n+1)}{n},\ q_2(VI) = \frac{\gamma v}{\alpha}.$$

In this case, investment levels by publisher P_2 remain constant across the two regimes. This implies that the utility of user types who are single-homers (in both the regimes) does not change. Instead, investment by the integrated retailer increases under vertical integration implying that total value creation through investments rises. A direct consequence is that the utility of multi-homing users increases. This increases their surplus and also encourages some single-homers (under vertical separation) to multi-home. This positive effect of vertical integration on total investment levels is reflected in the consumers' surplus. Specifically, we find that vertical integration increases consumer surplus.

$$CS(VS) - CS(VI) = -\frac{\gamma^2 v \left(2(1-c)n + \gamma^2(2n+1)v\right)}{2n^2} < 0.$$

Interestingly, the effect of vertical integration of total welfare is more nuanced.

$$W(VS) - W(VI) = \frac{\gamma^2 v \left(v(1-\gamma^2) - 2n(1+\gamma^2 v - c)\right)}{2n^2}.$$

To be precise, we find that total surplus increases under vertical integration if and only if the number of advertisers is large enough — i.e., $n > \max\{2, \frac{v(1-\gamma^2)}{2(1-c+\gamma^2 v)}\}$. The intuition for this result is as follows. When n is large, then investment of the integrated firm only changes marginally implying that the profit of the two firms is also impacted marginally. Note that consumer surplus is always higher under vertical integration. As a result, the positive effect of vertical integration on consumer surplus dominates any negative impact of increased cost of quality.

6 Remedies

The previous section has shown that the effect of vertical integration on consumer surplus and welfare can be positive or negative. The latter case is more likely when there is a substantial reduction in the quality provided by the independent publisher. It is therefore worthwhile to study the effects of possible regulatory interventions that may be adopted with the aim of restoring the VS market outcomes (short of breaking up the integrated firm).

Prohibiting data combination within ecosystems. Firms running ecosystems typically exploit combined datasets generated from different services they provide. We capture this behavior in the VI scenario by assuming that the integrated intermediary can exploit consumer data generated by P_1 in the sale of ad impressions on both publishers. Consider a regulatory intervention prohibiting data combination between I_1 and P_1. This would be consistent with the EU's Digital Market Act (DMA), which regulates the combination of users' data gathered from different services provided by the same firm. The act emphasizes that the ability of platforms to share data within the ecosystem should be conditional on user consent. To capture this regulation in our setting, suppose the data generated on P_1 cannot be used by I_1 to sell ads. However, the regulation does not forbid I_1 from collecting data on consumers visiting the publishers it serves as an intermediary. To the extent that P_1 exclusively uses I_1 as its intermediary, consumer data generated on P_1 would still be exclusively available to I_1 (regardless of user consent). Hence, the same equilibrium outcome as in Section 4.2 occurs. Hence, a policy focusing just on prohibiting data combination within an ecosystem would be ineffective.

Prohibiting exclusive access to ad inventories. Firms that offer multiple services within the same ecosystem tend to bundle these services together. For instance, any advertiser intending to place ads on Youtube must use Google's ad intermediation services. Accordingly, we assume that in the VI scenario only I_1 can distribute the impressions generated on P_1. A possible regulatory intervention could be to remove this exclusivity. Suppose however that the integrated firm maintains exclusive access to consumer data generated on P_1. This could be the result of a strict privacy policy by the ecosystem. For instance, Google recently launched the Privacy Sandbox initiative, with the intent of phasing-out third-party cookies. Currently, these cookies are essential for third-party intermediaries to provide targeted advertising and collect data on campaign performance. Some have expressed concerns that, by removing these capabilities for other intermediaries, the initiative may favor Google.

To analyze this scenario, suppose P_1 must now send bid requests to both intermediaries, but I_1 still has exclusive access to user data on P_1. Hence, unlike I_2, I_1 can inform advertisers about whether a user is a single- or a multi-homer, and observe which impressions a multi-homer is exposed to on P_1. Consider how Stage 3 of the model would change. Under the above assumptions, the willingness to pay of an advertiser for an impression sold by intermediary I_2 on publisher p would have a similar form as (6), because the intermediary could not distinguish between single- and multi-homers and avoid repetition of ads on the latter consumers. We would have

$$w_A^p = v\frac{D_p}{D_p + D_{12}} + v\frac{D_{12}}{D_p + D_{12}}\left(1 - \delta_{-p}(MH)\right) = v\left(1 - \frac{D_{12}}{D_p + D_{12}}\delta_{-p}(MH)\right) \quad p = 1, 2, \tag{21}$$

where $\delta_{-p}(MH)$ is the share of impressions on multi-homers the advertiser acquires on the other publisher. This share is not conditional on which intermediary the advertiser uses for such impressions, because I_2 cannot collect data generated on P_1 and is therefore unable to keep track of the impressions received by the given consumer on either publisher, regardless of which intermediary delivers such impressions.

The willingness to pay for impressions auctioned by I_1 on P_1 would be the same as in Section 4.2. However, now I_1 cannot fully observe which ads a multi-homing consumer is exposed to when visiting P_1, because those ads may be now served by I_2. Hence, we have $w_G^{p,MH} = v(1 - \delta^2_{-p})$ for any impression that is not repeated, where δ^2_{-p} is the share of impressions on multi-homers visiting $-p$ that the advertiser acquires via I_2.

Given the informational advantage retained by the integrated intermediary, it is intuitive that simply mandating access to P_1's ad inventory does not fundamentally change the market outcomes characterized in Section 4.2. As we show in Appendix A.3, the equilibrium is such that all advertisers buy their impressions from intermediary I_1, paying v for each impression. Moreover, given $\delta_{-p}(MH) = 1/n$, I_1 is still able to shade its bids to publisher P_2, who receives $v\left(1 - \frac{D_{12}}{D_2+D_{12}}\frac{1}{n}\right)$ for each impression.[23] Thus, the total revenue of all firms and their incentives to provide quality do not change.

Suppose now that the regulator imposes access to P_1's ad inventory to any intermediary *and* ensures that each intermediary serving such ads can collect consumer information on that domain. In this case, the intermediaries are effectively on a level playing field, just like in the VS scenario. Each can control the frequency of impressions on both publishers and thus maximise their value to the advertisers. It can easily be shown that, under these conditions, the same market equilibrium as in the VS scenario would emerge.

Proposition 5 *Prohibiting exclusive access to ad inventories within the integrated firm's ecosystem would induce the same market outcomes as with vertical separation only if the regulator also prohibits exclusive data access within the ecosystem. Prohibiting data combination or prohibiting exclusivity over inventories alone would not change the market outcomes compared to VI.*

Facilitating data sharing across intermediaries. A further possibility is to impose a regime of data sharing across intermediaries without removing I_1's exclusive access to P_1's ad inventories. Although this option would be harder to square with privacy protection than the options considered above, it would restore the separation market outcomes. To see why, consider that both intermediaries can avoid wasteful repetition of ads on multi-homers because they know which ad a multi-homer is exposed to. In Stage 3 of the model, therefore, each advertiser is willing to pay v for any impression on a single-homer. If the consumer is a multi-homer, the advertiser is willing to pay v if the impression is not repeated and zero otherwise, regardless of the intermediary. Given the number of advertisers is $n > 2$, the equilibrium price of any impression in a first-price auction must be $\overline{b}^i_p = v$, for any i and p. Therefore, we get $B^{p+} = v$ for any p, as in VS. The profit of each intermediary and publisher would also be the same. Each publisher would earn the same profit as in (4) and, thus, the equilibrium quality levels would be $q_p(VS)$. We can thus conclude the following.

[23] In this equilibrium, I_1 also pays a discounted amount for impressions to P_1, but given this is just an internal transfer for the integrated firm, its total profit remains the same as in the baseline model.

Proposition 6 *In presence of an integrated firm, mandating consumer data sharing across intermediaries induces the same market outcomes as with separation.*

Discussion. To summarize, our findings point to increased data sharing *across* ecosystems as a way of inducing same market outcomes as in the VS regime. Applying measures preventing data sharing *within* the ecosystem is not effective on its own. Similarly, it is not effective to impose non-exclusive provision of ad inventories if the ecosystem retains exclusive control of the consumer data generated on its own websites.

Our findings are consistent with the provisions included in the DMA regarding the prevention of gatekeepers from unfairly benefiting from their data advantage and on the discussion on data interoperability to boost the contestability of markets.[24] However, our analysis also highlights the existence of trade-off between the efficiency of the ad market and the protection of consumer privacy, which could be significantly more difficult with greater data sharing across ecosystems. This observation is consistent with the ongoing debate on the possible unintended consequences of privacy regulation such as the GDPR. Furthermore, the analysis also lends support to concerns that ecosystems such as Google may support the adoption of stricter privacy controls as a means to maintain an informational and competitive advantage versus third-party competitors. More precisely, our analysis suggests that stricter privacy rules, limiting data access to multiple intermediaries, can result in a higher adtech tax, to the benefit of large integrated platforms and to the detriment of third-party publishers and their incentive to invest in content quality.

7 Conclusions

Intermediaries play a key role in the online advertising market, by connecting digital publishers to advertisers. Recent market studies have raised concerns about the dominant position of one such intermediary (Google), which is reinforced by the firm being integrated in the market for devices, browsers and digital content. In this paper, we proposed a model where two competing intermediaries sell the ad impressions generated by two publishers, comparing the scenario where these firms are separate to that where an intermediary and a publisher are integrated. We found that the integrated firm benefits from policies that limit the intermediaries' ability to track consumers across outlets, while the other firms are worse off. Moreover, integration enables an intermediary to squeeze the profit of the independent publisher, by capturing a larger share of the rev-

[24] The DMA states that "to prevent gatekeepers from unfairly benefitting from their dual role, it is necessary to ensure that they do not use any aggregated or non-aggregated data, which could include anonymised and personal data that is not publicly available to provide similar services to those of their business users." We may think of intermediaries as business users of browsers. Any privacy policies allowing the integrated intermediary to use data not available to rival intermediaries should be audited.

enue generated from its ad inventory on multi-homing consumers. Therefore, the analysis provides a foundation for the adtech tax, suggesting that this tax is directly related to vertical integration and to the informational advantage conferred to the integrated firm by consumer and advertiser multi-homing. Finally, integration gives the firm stronger incentives to invest in content quality, as doing so increases the share of multi-homers. On the contrary, the independent publisher's incentives to provide quality decrease.

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A Proofs Omitted in the Text

A.1 Proof of Lemma 2

Consider advertiser k's bidding strategy. We denote advertiser k's willingness-to-pay for an impression on publisher p delivered by intermediary i as $w^i_{p,k}$. As explained in the text, if the impression falls on a single-homer, the advertiser's willingness to pay for the impression is v (as the number of advertisers is $n \geq 3$). Hence, the equilibrium price for an impression on a single-homer must be v. If the impression falls on a multi-homer, then $w^i_{p,k} = v(1 - \delta^{-i}_{-p,k})$, where $\delta^{-i}_{-p,k}$ is the share of impressions advertiser k acquires through intermediary $-i$ on publisher $-p$. There is a positive probability of repetition only if two different intermediaries sell the two impressions on the same multi-homer on two different publishers.

We thus focus on the bidding strategies for impressions on multi-homers. To characterize them, we have to establish which values of $\delta^i_{p,k}$ can emerge in any equilibrium of the subgame. For ease of exposition, we consider the case where $n = 3$. Most of our arguments apply with any larger number of advertisers and we discuss relevant differences when necessary.

Let (a, b, c) be the set of advertisers. Focus on the relation between $\delta^i_{p,k}$ and the willingness to pay for impressions on publisher $-p$ sold by intermediary $-i$. The following cases are possible (we present here just one of the possible permutations of the advertisers without loss):

- Case A: if $\delta^i_{p,c} > \delta^i_{p,b} > \delta^i_{p,a}$, the willingness-to-pay for each impression on multi-homers on $-p$ distributed by $-i$ are $w^{-i}_{-p,c} = v\left(1 - \delta^i_{p,c}\right) < w^{-i}_{-p,b} = v\left(1 - \delta^i_{p,b}\right) < w^{-i}_{-p,a} = v\left(1 - \delta^i_{p,a}\right)$. Hence, a outbids the others for all such impressions, so $\delta^{-i}_{-p,c} = \delta^{-i}_{-p,b} = 0 < \delta^{-i}_{-p,a} = 1$.

- Case B: if $\delta^i_{p,c} > \delta^i_{p,b} = \delta^i_{p,a}$, the willingness-to-pay for each impression on multi-homers on $-p$ distributed by $-i$ are $w^{-i}_{-p,c} = v\left(1 - \delta^i_{p,c}\right) < w^{-i}_{-p,b} = v\left(1 - \delta^i_{p,b}\right) = w^{-i}_{-p,a} = v\left(1 - \delta^i_{p,a}\right)$. Hence, a and b outbid c for all such impressions, i.e. $\delta^{-i}_{-p,b} = \delta^{-i}_{-p,a} = 1/2 > \delta^{-i}_{-p,c} = 0$.

- Case C: if $\delta^i_{p,c} = \delta^i_{p,b} = \delta^i_{p,a}$, the willingness-to-pay for each impression on multi-homers on $-p$ distributed by $-i$ are $w^{-i}_{-p,b} = w^{-i}_{-p,a} = w^{-i}_{-p,c}$. The advertisers place identical bids for such impressions, so $\delta^{-i}_{-p,b} = \delta^{-i}_{-p,a} = \delta^{-i}_{-p,c} = 1/3$.

In equilibrium, the bidding strategy on publisher $-p$ through intermediary $-i$ should be compatible with that on publisher p through intermediary i (and the ensuing share $\delta^p_{i,k}$). Suppose case A applies. Because $\delta^{-i}_{-p,c} = \delta^{-i}_{-p,b} = 0 < \delta^{-i}_{-p,a} = 1$ by the same reasoning as above we must have $\delta^i_{p,c} = \delta^i_{p,b} = 1/2 > \delta^i_{p,a} = 0$. This outcome is inconsistent with $\delta^i_{p,c} > \delta^i_{p,b} > \delta^i_{p,a}$, so we can disregard this case.

Suppose Case C applies. Because $\delta^{-i}_{-p,b} = \delta^{-i}_{-p,a} = \delta^{-i}_{-p,c} = 1/3$, by the same reasoning as above we must have $\delta^{i}_{p,b} = \delta^{i}_{p,a} = \delta^{i}_{p,c} = 1/3$. Hence, each advertiser's willingness-to-pay for each impression on p sold by i is $v\,(1 - 1/3) = 2v/3$. So $\overline{b}^{i}_{p} = \overline{b}^{-i}_{p} = 2v/3$. Given the bids placed by the rivals, in any of these candidates each advertiser can deviate by bidding $2v/3+\varepsilon$ (with $\varepsilon > 0$ and arbitrarily small) for all impressions on one intermediary (winning them all) and zero for all impressions on the other. The advertiser would earn a strictly positive profit from these impressions (each would generate a return of v, since impressions cannot be repeated). The advertiser earns zero in the equilibrium candidate, so the deviation is profitable. We can therefore disregard this case in the following.

Suppose Case B applies. Because $\delta^{-i}_{-p,b} = \delta^{-i}_{-p,a} = 1/2 > \delta^{-i}_{-p,c} = 0$, by the same reasoning as above we must have $\delta^{i}_{p,c} = 1 > \delta^{i}_{p,b} = \delta^{i}_{p,a} = 0$. Hence, a and b's willingness-to-pay is v for impressions auctioned by $-i$ on $-p$, while c's is 0. Hence $\overline{b}^{-i}_{-p} = v$. Advertiser c's willingness to pay for impressions sold by i on p is v, while a and b's willingness to pay is $v\,(1 - 1/2) = v/2$. So $\overline{b}^{i}_{p} = v/2$. In this case, the profits of a and b are zero, while c's profits are $(v - \frac{v}{2}) = \frac{v}{2}$.

We have thus identified equilibrium candidates in case B. This implies that there cannot be equilibria where an advertiser buys ads on both publishers from two different intermediaries. The equilibrium candidates are such that advertisers use a single intermediary and place ads on both publishers, or place ads on a single publisher, but use multiple intermediaries. We now examine these candidates equilibria.

Consider first an equilibrium candidate where all advertisers buy their impressions from a single intermediary, possibly on different publishers. For instance, suppose a and b only buy ads on I_2, whereas c buys them on I_1. Hence, $\delta^{1}_{p,c} = 1 > \delta^{1}_{p,b} = \delta^{1}_{p,a} = 0$. Moreover, $\delta^{2}_{p,c} = 0 < \delta^{2}_{p,b} = \delta^{2}_{p,a} = \frac{1}{2}$ because intermediary I_2 avoids that impressions sold to a and b are repeated on publisher p. Hence, the bid on intermediary I_2 for an impression on publisher p is $\overline{b}^{2}_{p} = v$, because there are two advertisers with a willingness to pay equal to v on I_2. Also, the bid on intermediary I_1 for an impression on publisher p is also $\overline{b}^{1}_{p} = v$, because c has a willingness to pay equal to v, and knows that the bid of the other advertisers a and b on the other intermediary I_2 is v. Hence, both intermediaries send a bid $\overline{B}_p = v$, and each gain half of the total impressions. c buys all impressions sold on I_1 making a profit equal to 0, while a and b buy half of the impressions sold by I_2, making a profit equal to 0. No advertiser can profitably deviate because impressions are sold at v. Hence, this candidate is an equilibrium.

Consider now an equilibrium candidate where all advertisers buy their impressions from a single publisher and use multiple intermediaries. To start, consider the case with $n = 3$. For instance, suppose a and b only buy ads on P_2, whereas c buys them on P_1. That is $\delta_{1,c} = 1, \delta_{2,a} = \delta_{2,b} = 1/2$ (we drop the superscript i as we refer to shares of impressions on publishers regardless of the intermediary). In this candidate, $\overline{b}^{i}_{1} = v$, $\forall i$, while $\overline{b}^{i}_{2} = v/2$. In this case, equilibrium profits of advertiser c is $\frac{v}{2}$, while profits of advertisers a and b are 0.

Consider advertisers a or b (say, a) deviates bidding $\frac{v}{2}+\epsilon$ on publisher P_1. Advertiser a would outbid c and win all impressions on P_1, making a positive profits. Hence, this deviation is profitable.

Now, consider the case with $n = 4$. Suppose a and b only buy ads on P_2, whereas c and d buy them on P_1. Then we have for each advertiser k that $\delta_{p,k} = 1/2$ (we drop the subscript i as we refer to shares of impressions on publishers regardless of the intermediary). In this candidate, $\overline{b}_p^i = v$, $\forall i$ and $\forall p$. In this case, equilibrium profits of each advertiser are 0. Hence, advertisers are earning the same profits as under the equilibrium where advertisers single-home on each intermediary. Considering our refinement of one-stop shop campaigns, we can exclude this candidate.

Then, the equilibrium candidate where all advertisers buy on the same publisher can be ruled out because impressions on multi-homers on the other publisher would be sold at v/n, while each advertiser would pay v for the impressions it acquires, making zero profit. Hence, each advertiser could profitably deviate by bidding $v/n+\epsilon$ for the impressions on the other publisher and zero for all other impressions.

Consider now a candidate such that one advertiser only buys impressions on one publisher but on multiple intermediaries, while the other two buy their impressions on both publishers but from a single intermediary. Suppose, for example that $\delta_{1,c} > \delta_{2,c} = 0$, i.e. advertiser c does not place any ads on P_2, while $\delta^1_{2,b} = \delta^1_{2,a} = \delta^1_{1,b} = \delta^1_{1,a} = 0$, i.e., a and b only place ads via I_2. The willingness to pay for any (non-repeated) impression on either P_2 or P_1 auctioned by I_2 is v. Hence all impressions on P_1 are sold at v. Indeed, the bid for impressions on P_1 is v, since at least two advertisers have a willingness to pay of v for them, hence $\overline{b}_2^1 = \overline{b}_2^2 = v$. Instead, $\overline{b}_2^1 = \overline{b}_2^2 = v/2$. Advertiser c pays v for all its impressions and can thus profitably deviate by bidding $v/2+\epsilon$ for impressions on P_2 and zero for impressions on P_1. If $n \geq 4$, there may exist equilibria where two advertisers buy ads only via I_2, while the other advertisers buy ads only on one publisher (though not all the same one) and from different intermediaries. These equilibria would be such that for each impression there are at least two advertisers willing to bid v, so that would also be the price of each impression. No advertiser could profitably deviate but the market outcome would be identical to the one we consider in the text. Hence, considering our refinement of one-stop shop campaigns, we can exclude this candidate.

Finally, there is no equilibrium where all advertisers buy all their ads via the same intermediary. Suppose this was the case and say this intermediary is I_1. Each advertiser would be willing to pay v for each (non-repeated) impression, so v is their price. No advertiser would make a profit and each would get 1/3 of all the impressions. An advertiser could deviate by bidding v for all (non-repated) impressions auctioned by I_2. This deviation would result in I_2 distributing 1/2 of the impressions, all to the deviating advertiser. Hence, the advertiser would acquire strictly more impressions while making the same profit, which is preferable by one of the refinements we assumed.

A.2 Proof of Lemma 5 (bids under vertical integration)

Given the willingness to pay for impressions outlined in Section 4.2, an impression on a single-homer auctioned by I_1 must be sold to advertisers at v (on either publisher), because all advertisers are willing to pay v. Similarly, an impression on a multi-homer sold by I_1 on P_2 must be sold at price v to advertisers, because, given $n \geq 3$ advertisers, there must be at least two of them willing to pay v (i.e., those who are not sending an impression to the same consumer on P_1). Hence, intermediary I_1 can raise its bid for an impression on a single- and multi-homer on P_2 up to v. Hence, intermediary I_2 can win an impression on a consumer (a single- or a multi-homer) on P_2 only if the winning bid received from the advertisers equal v.

Let $k \in (a, b, c)$ be the set of advertisers (our reasoning holds for every $n \geq 3$). We denote as $w^i_{p,k}$ the willingness to pay of advertiser k for impressions on p sold by i and as $\delta^i_{p,k}$ being the share of impressions on multi-homers bought by advertiser k on publisher p by intermediary i.

The willingness to pay of advertiser k for impressions on P_2 sold by I_2 is $w^2_{2,k} = v\left(1 - \frac{D_{12}}{D_2+D_{12}}\delta^1_{1,k}\right)$, and depends on the share $\delta^1_{1,k}$ of impressions on multi-homers bought by k on P_1 through I_1 (see (6), noticing that in the text we abstract from subscript k to simplify notation). Hence, this willingness to pay is equal to v if and only if $\delta^1_{1,k} = 0$.

When bidding for impressions on multi-homers on P_1 sold by I_1 and on impressions auctioned by I_2 (which cannot be distinguished between single- and multi-homers) on publisher P_2, the advertisers' willingness to pay depends therefore on the share of impressions on multi-homers they buy on the other platform. The relevant cases are as follows ones (we present here just one of the possible permutations of the advertisers without loss):

- Case A: if $\delta^1_{1,c} = \delta^1_{1,b} > \delta^1_{1,a} = 0$, the willingness-to-pay for each impression on P_2 distributed by I_2 are $w^2_{2,a} = v$ and $w^2_{2,k} = v\left(1 - \frac{D_{12}}{D_2+D_{12}}\delta^1_{1,k}\right)$ for $k \in \{b, c\}$. Hence, a has a higher willingness to pay than b and c for all such impressions, and can acquire all impressions on P_2 sold by I_2.

- Case B: if $\delta^1_{1,c} = 1 > \delta^1_{1,b} = \delta^1_{1,a} = 0$, the willingness-to-pay for each impression on a consumer on P_2 distributed by I_2 are $w^2_{2,a} = w^2_{2,b} = v > w^2_{2,c} = v\left(1 - \frac{D_{12}}{D_2+D_{12}}\delta^1_{1,c}\right)$. Hence, a and b have a higher willingness to pay than c for all such impressions, acquiring half of the impressions on P_2 sold by I_2.

- Case C: if $\delta^1_{1,c} = \delta^1_{1,b} = \delta^1_{1,a} = 0$, the willingness-to-pay for each impression on P_2 distributed by I_2 are $w^2_{2,k} = v \ \forall k$.

Note that, in all cases, intermediaries I_2 and I_1 receive a top bid equal to v for each impression on P_2 by the winning advertiser, hence they both bid v to the publisher. Each intermediary is therefore awarded half of the impressions on publisher P_2.

In case A, advertiser a has a willingness to pay equal to v for an impression on P_2, irrespective of whether it is auctioned by I_2 or I_1. This is because the advertiser is not acquiring any impression on multi-homers on P_1. Note that, even if the other advertisers have lower willingness than v, the only chance for a to win impressions on I_2 is to bid v (because the winning bid on I_1 for such impressions is v). If a bids v, then intermediary I_2 wins one half of the impressions on P_2, and the other half is won by I_1. Advertiser a thus wins all the impressions on P_2 sold by I_2 and $\frac{1}{3}$ of impressions on multi-homers on P_2 sold by I_1 (because all advertisers bid v for those impressions, as long as they are not repeated), zero impressions on multi-homers on P_1 sold by I_1, and $\frac{1}{3}$ of the impressions on single-homers. Advertiser a would thus make zero profits. Suppose now that the advertiser deviates, and starts bidding for impressions on multi-homers sold by I_1. In this case, it bids v for impressions on multi-homers on publisher P_1 sold by intermediary I_1 and on multi-homers on P_2 sold by intermediary I_1. In this case, it makes zero profits, it is awarded $\frac{1}{3}$ of the impressions on P_1 sold by I_1, $\frac{1}{3}$ of the impressions on P_2 sold by I_1, and zero impressions sold by I_2. Hence, it has same profits and same number of impressions as in the equilibrium candidate. However, this deviation is profitable given the one-stop shopping refinement.

In case B, advertiser $k \in (a, b)$ bids v for an impression on P_2, irrespective of whether it is auctioned by I_2 or I_1. This is because these advertisers do not acquire any impression on multi-homers on P_1. Each advertiser $k \in (a, b)$ wins $\frac{1}{2}$ of the impressions on multi-homers on P_2 sold by I_2, $\frac{1}{3}$ of the impressions on multi-homers on P_2 sold by I_1, zero impressions on multi-homers on P_1 and $\frac{1}{3}$ of the impressions on single-homers. The advertisers also make zero profits, as they pay v for every impression acquired. Consider the following deviation for one advertiser in this set, say a. Consider that advertiser a deviates, and starts bidding for impressions on multi-homers sold by I_1. In this case, it bids v for impressions on multi-homers on publisher P_1 sold by intermediary I_1 and on multi-homers on P_2 sold by intermediary I_1, while it bids $v\left(1 - \delta^1_{1,a}\right)$, with $\delta^1_{1,c} > 0$. In this case, it makes zero profits, it is awarded $\frac{1}{2}$ of the impressions on multi-homers on P_1 sold by I_1, $\frac{1}{3}$ of the impressions on multi-homers on P_2 sold by I_1, $\frac{1}{3}$ of the impressions on single-homers, and zero impressions sold by I_2. Hence, it has the same profits and same number of impressions as in the equilibrium candidate. However, this deviation is profitable given the one-stop shopping refinement.

In case C, advertiser $k \in (a, b, c)$ bids v for an impression on P_2, irrespective of whether it is sold by I_2 or I_1. This is because none of them is acquiring any impression on P_1. Each advertiser wins $\frac{1}{3}$ of the impressions on P_2 sold by I_2, $\frac{1}{3}$ of the impressions on P_2 sold by I_1, and zero impressions on multi-homers on P_1 sold by I_1, making zero profits. Consider the following deviation for one advertiser in this set, say a. Suppose the advertiser deviates by bidding v for impressions on multi-homers on publisher P_1 sold by intermediary I_1 and on multi-homers on P_2 sold by intermediary I_1, while it bids less than v for impressions on P_2 sold by I_2. In this case, it makes zero profits, and it is awarded all the impressions on multi-homers on publisher P_1 sold by I_1, obtaining the same profits as in the equilibrium candidate but a bigger share of impressions. Hence, this deviation is profitable because we assumed the advertiser prefers to buy a bigger share of impressions given a level of profits.

Hence, we conclude there is no equilibrium such that at least two advertisers do not bid on impressions on P_1 sold by intermediary I_1.

Assume now all the advertisers bid v for any (non-repeated) impression on multi-homers on P_1. There is no profitable deviation, hence this is an equilibrium. In this case, the highest bid I_2 collects is equal to the willingness to pay of advertisers who buy impressions on multi-homers on P_1, that is $v\left(1 - \frac{D_{12}}{D_2+D_{12}}\delta^1_{1,c}\right)$, where $\delta^1_{1,c} = \frac{1}{n}$. In a first price auction, this is also the amount that I_1 bids to P_2 to acquire all its impressions.

A.3 Characterizing the equilibrium without exclusive access to P_1's ad inventory in Section 6

Given the willingness to pay for impressions outlined in the text, an impression on a single-homer auctioned by I_1 must be sold at v (on either publisher). This also implies that $\overline{B}^1_p(SH) = v$, so that I_2 can never outbid I_1 for such impressions, i.e., all advertisers acquire a positive amount of these impressions via I_1 in any equilibrium.

Our next step is to establish that in any equilibrium, no advertiser can acquire impressions via I_2 on both publishers. Recall the expression for w^2_p in (21). Suppose there is a set of advertisers that acquire a positive quantity of these impressions and a set of advertisers who do not. For the former set, $\delta_p(MH)$ exceeds that of the other advertisers by definition, which in turn means that w^2_{-p} for the first set of advertisers must be smaller than for the second set. It follows that the first set of advertisers do not acquire any impression via I_2 on publisher $-p$.

Suppose now that all advertisers acquire impressions on p via I_2. The advertisers must have the same value of w^2_p, which must also equal to their price in a first-price auction. Hence, none of these impressions generates any profit for the advertisers. Given $\delta^2_p = 1/n > 0$ for all advertisers, the price of impressions on multi-homers sold by I_1 on publisher $-p$ must be $v(1-1/n)$. Hence, any advertiser can profitably deviate from this equilibrium candidate by bidding zero for the impressions sold by I_2 and $v(1-1/n)+\epsilon$ for the impressions on multi-homers sold by I_1 on $-p$ (which would be worth v to the advertiser). The advertiser would acquire all such impressions and make more profit.

We have therefore established that if an equilibrium exists such that one or more advertisers acquire impressions via I_2, these impressions cannot take place on both publishers and must be such that some advertisers place ads on P_2 and others place ads on P_1. Suppose now, without loss, that more than one advertiser acquires impressions on P_2. The two (or more) such advertisers must have equal willingness to pay for these impressions and the equilibrium price must equal this willingness to pay. Hence, they make zero profits from these impressions. Given these advertisers must also acquire some impressions on single-homers via I_1, as stated above, they can make at least as much profit by deviating and placing winning bids only for impressions sold by I_1. This deviation is preferable by assumption, since it would imply a "one-stop" campaign.

We can therefore conclude that, under our assumptions, there is no equilibrium where advertisers acquire impressions via I_2. That is, $\delta_p^2 = 0, \forall p$ in equilibrium. Therefore, each advertiser has a willingness to pay of v for any non-repeated impression sold by I_1. The equilibrium prices are thus as characterized in the text.

A.4 Reservation price

Now we change the model described in Section 3, introducing a preliminary stage 0, where v is drawn from a continuous distribution $F(v)$ with support $[0, \bar{v}]$. We assume this distribution is common knowledge. v is known by the advertisers, but it is not observed by the publishers. At stage 1, the publisher can impose a reservation price r for its impressions.

In the VS scenario, the publishers have no use for the reservation price given they obtain a price of v for every impression.

Consider now the VI scenario and suppose P_2 can impose a reservation price r for its impressions. Note that this price cannot be conditioned on the realization of v, which is unobservable to the publisher.

Given v and the equilibrium price of impressions in the baseline model, $\overline{B}_2(VI) = v\left(1 - \frac{D_{12}}{D_2+D_{12}}\frac{1}{n}\right)$, the reservation price will be binding if and only if $r > v(1-m)$, where $m \equiv \left(\frac{D_{12}}{D_2+D_{12}}\frac{1}{n}\right)$ for convenience. Specifically, the price of each impression shown by P_2 will be $v(1-m)$ if $r \leq v(1-m)$, and it will be r if $v \geq r > v(1-m)$. No impression will be sold if $r > v$. Therefore, given r, P_2's expected revenue is as follows:

$$R_2 = \left(\int_r^{\frac{r}{(1-m)}} r dF(v) + (1-m)\int_{\frac{r}{1-m}}^{\bar{v}} v dF(v)\right)(D_2 + D_{12}), \quad if \quad \frac{r}{1-m} < \bar{v},$$

$$R_2 = \left(\int_r^{\bar{v}} r dF(v)\right)(D_2 + D_{12}), \quad if \quad \frac{r}{1-m} \geq \bar{v}.$$

As we have seen in the baseline model, without the reservation price, the publisher expects to pay an adtech tax of vm for every impression. The above expressions show that the reservation price may reduce, but not eliminate, this tax. The tax equals $v - r$ for any impression whenever $r \leq v < r/(1-m)$ and still equals vm whenever $\bar{v} \geq v \geq r/(1-m)$.

The profit-maximizing value of r for P_2 depends on the distribution $F(v)$. While we do not establish this price, we note that for many such distributions the optimal reservation price will be such that $\bar{v} \geq v \geq r/(1-m)$. Note also that whenever the optimal r is such that $\frac{r}{1-m} < \bar{v}$, the expected revenue of P_2 per impression is decreasing in $m = \frac{D_{12}}{D_2+D_{12}}\frac{1}{n}$, just like in the baseline model. Indeed, taking the derivative of $R_2(r)$ with respect to m we find

$$\frac{\partial R_2}{\partial m} = \frac{r}{(1-m)^2} rF\left(\frac{r}{(1-m)}\right) - \int_{\frac{r}{(1-m)}}^{\bar{v}} vdF(v) +$$
$$-(1-m)\frac{r}{(1-m)^2}\frac{r}{(1-m)}F\left(\frac{r}{(1-m)}\right) = -\int_{\frac{r}{(1-m)}}^{\bar{v}} vdF(v).$$

It follows that similar incentives as in the baseline model apply to both publishers with regards to the choice of quality.

B Analysis of the Examples

B.1 Independent consumer preferences

We assume that the consumers are distributed uniformly with respect to their value for the content offered by the publishers. We employ a uniform distribution with the unit support — i.e., $u_1 \sim \mathcal{U}[0,1]$ and $u_2 \sim \mathcal{U}[0,1]$. Under these assumptions, we are able to make informed and clear cut presentation of the impact of vertical integration on consumer surplus and total welfare.
The associated single-homing consumer demand at each publisher p and the multi-homing demand is

$$D_p(q_p, q_{-p}) = (1-c+\gamma q_p)(c-\gamma q_{-p}), \quad D_{12}(q_1, q_2) = (1-c+\gamma q_1)(1-c+\gamma q_2) \text{ for } p \in \{1,\ 2\}.$$

Vertical Separation. The equilibrium quality levels at the publisher p is given as $q_p(VS) = v\gamma$, for $p \in \{1,\ 2\}$. The ensuing single-homing and multi-homing demands are respectively given as follows.

$$D_p(VS) = \left(1-c+v\gamma^2\right)\left(c - v\gamma^2\right),\ D_{12}(VS) = \prod_{i=1,2}\left(1-c+v\gamma^2\right) \text{ for } p \in \{1,\ 2\}.$$

The equilibrium profit of publisher p and the advertisers is given as

$$\pi_p(VS) = \frac{v(2(1-c)+v\gamma^2)}{2}, \qquad \pi_{Ad}(VS) = 0, \text{ for } p \in \{1,\ 2\}.$$

The profit of the intermediaries is $\pi_i(VS) = 0$ for $i \in \{1,\ 2\}$. Consumer surplus is given as $CS(VS) = (1 + v\gamma^2 - c)^2$. Total welfare is then $W(VS) = CS(VS) + \pi_1(VS) + \pi_2(VS) = (1-c)^2 + v(1+\gamma^2)(2(1-c) + v\gamma^2)$.

Vertical Integration. The equilibrium quality levels at the integrated publisher P_1 and the independent publisher P_2 is respectively given as

$$q_1(VI) = \frac{\gamma v\left(n(n+1-c) + \gamma^2 v(n+c-1)+\right)}{n^2 + v^2\gamma^4}, q_2(VI) = \frac{\gamma v\left(n(n+c-1) - \gamma^2 v(n+1-c)\right)}{n^2 + v^2\gamma^4}.$$

The associated single-homing and multi-homing demands are

$$D_1(VI) = (1 + \gamma q_1(VI) - c)(c - \gamma q_2(VI)), D_2(VI) = (1 + \gamma q_2(VI) - c)(c - \gamma q_1(VI)),$$

and

$$D_{12}(VI) = (1 + \gamma q_1(VI) - c)(1 + \gamma q_2(VI) - c).$$

The profit of the integrated firm P_1, *the independent publisher* P_2 and the advertisers is respectively given as

$$\pi_1(VI) = v(D_1(VI) + D_{12}(VI)) + \frac{vD_{12}(VI)}{n} - \frac{k_1(q_1(VI))^2}{2},$$

$$\pi_2(VI) = v(D_2(VI) + D_{12}(VI)) - \frac{vD_{12}(VI)}{n} - \frac{k_2(q_2(VI))^2}{2} \text{ and, } \pi_{AD}(VI) = 0.$$

Consumers surplus under vertical integration is given as

$$CS(VI) = \frac{n^2\left(1 + v\gamma^2 - c\right)^2}{n^2 + v^2\gamma^4}.$$

Total welfare is then $W(VI) = CS(VI) + \pi_1(VI) + \pi_2(VI)$.

Welfare implications of vertical integration. The profit of the vertically integrated firm is higher than the profit of the independent publisher. This is straightforward as the vertically integrated firm is able to skim off a portion of the revenues to the independent publisher via "Bid-Shading".

Taking the difference of the consumer surplus in the two cases yields

$$CS(VS) - CS(VI) = \frac{\gamma^4 v^2\left(1 + \gamma^2 v - c\right)^2}{n^2 + \gamma^4 v^2} > 0.$$

The above difference is always positive implying that vertical integration hurts consumers

vis-á-vis vertical separation. Thus, we show that in this example consumers are better off under vertical separation.
Taking the difference of the total surplus in the two cases yields

$$W(VS) - W(VI) = \frac{v^2\gamma^2 (1+\gamma^2)(1+v\gamma^2-c)^2}{n^2+v^2\gamma^4} > 0.$$

The above expression is always positive. Thus, we show that total welfare falls after a vertical integration.

B.2 Negatively correlated preferences.

We assume that the consumers are distributed uniformly with respect to their value for the content offered by the publishers. We employ a uniform distribution with the unit support for the preference parameter $u_1 \sim \mathcal{U}[0,1]$. Further, we assume negative correlation between u_2 and u_1 and employ a simple transformation with $u_2 = 1 - u_1$. The consumer segmentation in this case can then be represented as in the following figure.

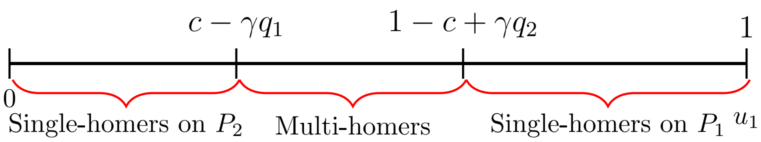

Figure 7: Single and multi-homing consumers.

Thus, the associated single-homing consumer demand at each publisher p and the multi-homing demand is

$$D_1 = c - \gamma q_2,\ D_2 = c - \gamma q_1,\ D_{12}(q_1, q_2) = 1 - c + \gamma q_2 - (c - \gamma q_1).$$

Vertical Separation. The equilibrium quality levels at the publisher p and the ensuing single-homing and multi-homing demands are respectively given as follows.

$$q_p(VS) = v\gamma,\ D_p(VS) = c - v\gamma^2,\ D_{12}(VS) = 1 + 2(v\gamma^2 - c).$$

The equilibrium profit of publisher p, advertisers and the advertising network p is given as

$$\pi_p(VS) = \frac{v(2(1-c)+v\gamma^2)}{2}, \qquad \pi_{Ad}(VS) = 0, \qquad \pi_{AN}(VS) = 0 \text{ for } p \in \{1,\ 2\}.$$

Consumer surplus in our setting is given as $CS(VS) = (1 - c + v\gamma^2)^2$. Total welfare is then $W(VS) = CS(VS) + \pi_1(VS) + \pi_2(VS) = (1-c)^2 + v(1+\gamma^2)(2(1-c)+v\gamma^2)$.
Vertical Integration. The equilibrium outcome under vertical integration is as follows.

The equilibrium quality levels at the integrated publisher 1 and the independent publisher P_2 is respectively given as $q_1(VI) = \frac{(n+1)v\gamma}{n}$ and $q_2(VI) = \frac{(n-1)v\gamma}{n}$. The associated single-homing and multi-homing demands are

$$D_1(VI) = c - \gamma q_2(VI), D_2(VI) = c - \gamma q_1(VI), D_{YC}(VI) = 1 + 2(v\gamma^2 - c).$$

The profit of the integrated publisher 1 is

$$\pi_1(VI) = \frac{v\left((1-c)(n+1) - c + v\gamma^2(n+3)\right)}{n} - \frac{\left(\frac{(n+1)v\gamma}{n}\right)^2}{2}.$$

The profit of *the independent publisher* P_2 and the advertisers is respectively given as

$$\pi_2(VI) = \frac{v\left((1-c)(n-1) + c + v\gamma^2(n-3)\right)}{n} - \frac{\left(\frac{(n-1)v\gamma}{n}\right)^2}{2}, \ \pi_{AD}(VI) = 0.$$

Consumers surplus under vertical integration is given as

$$CS(VI) = \frac{n^2\left(1 + \gamma^2 v - c\right)^2 + \gamma^4 v^2}{n^2}.$$

Total welfare is then $W(VI) = CS(VI) + \pi_1(VI) + \pi_2(VI)$.

Welfare implications of vertical integration. The profit of the vertically integrated firm is higher than the profit of the independent publisher.

Taking the difference of the consumer surplus in the two cases yields

$$CS(VS) - CS(VI) = -\frac{v^2\gamma^4}{n^2} < 0.$$

The above expression is always negative in the relevant parameter range implying that vertical integration benefits consumers. Thus, we show that in this example consumers are better off under vertical integration.

Taking the difference of the total surplus in the two cases yields

$$W(VS) - W(VI) = \frac{v^2\gamma^2(1-\gamma^2)}{n^2} > 0.$$

The above expression is positive when $\gamma < 1$ and negative otherwise.

B.3 Positive correlation of preferences.

We assume that the consumers are distributed uniformly with respect to their value for the content offered by the publishers. Further, these values are positively correlated such that $u_2 = \alpha u_1$ with $u_1 \sim [0,1]$ and $u_2 \sim [0,\alpha]$ with $\alpha \in [0,1]$. Thus, utility of consumers when visiting P_2 can be appropriately modified.

For a complete analysis of this extension, two cases must be considered — i.e., (i) $\bar{u}_1 \leq \bar{u}_2$ and (ii) $\bar{u}_1 > \bar{u}_2$ where the definition of $\bar{u}_1$ and $\bar{u}_2$ is explained below. This is because the demand structure is different in the two cases.

(i) Case $\bar{u}_1 \leq \bar{u}_2$. Consumers visit content providers when they receive positive utility. Thus, consumers participate on P_1 and P_2 respectively when

$$V_1 \geq 0 \implies u_1 \geq \bar{u}_1 := c - \gamma q_1 \text{ and } V_2 \geq 0 \implies u_1 \geq \bar{u}_2 := \frac{c - \gamma q_2}{\alpha}.$$

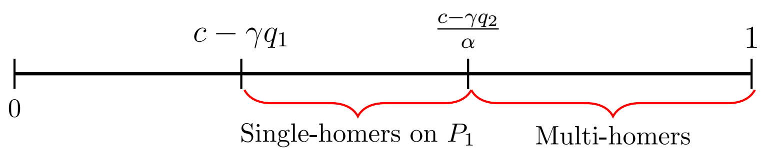

Figure 8: Single and multi-homing consumers.

The associated demands are $D_1 = \frac{c-\gamma q_2}{\alpha} - (c - \gamma q_1)$ and $D_2 = 0$ and $D_{12} = 1 - \frac{c-\gamma q_2}{\alpha}$.
Vertical Separation. The equilibrium quality levels at the publisher P_1 and P_2 are respectively given as follows. $q_1(VS) = v\gamma$ and $q_2(VS) = \frac{v\gamma}{\alpha}$ The equilibrium profit of publishers P_1 and P_2 is $\pi_1(VS) = \frac{2v(1-c)+v\gamma^2}{2}$ and $\pi_2(VS) = \frac{v(v\gamma^2+2\alpha(\alpha-c))}{2\alpha^2}$. The profit of advertisers and the intermediary p is zero. Consumer surplus in this setting is

$$CS(VS) = \frac{\alpha^2\left(\alpha^2 + \alpha + c^2 + \alpha(c-4)c\right) + 2\alpha\gamma^2 v\left(\alpha(\alpha+1) - \left(\alpha^2+1\right)c\right) + \left(\alpha^3+1\right)\gamma^4 v^2}{2\alpha^3}.$$

Welfare is given as $W(VS) = CS(VS) + \sum_{p=1,2} \pi_p(VS)$.
Vertical Integration. The equilibrium quality levels at the integrated publisher P_1 and the independent publisher P_2 is respectively given as $q_1(VI) = v\gamma$, $q_2(VI) = \frac{(n-1)v\gamma}{\alpha n}$.

The profit of the integrated publisher 1, the independent publisher P_2 and is respectively given as $\pi_1(VI) = \frac{v\left(2\alpha n(\alpha(n+1)-c(\alpha n+1))+\gamma^2 v\left(n\left(\alpha^2 n+2\right)-2\right)\right)}{2\alpha^2 n^2}$, $\pi_2(VI) = \frac{(n-1)v\left(2\alpha n(\alpha-c)+\gamma^2(n-1)v\right)}{2\alpha^2 n^2}$.
Consumers surplus under vertical integration is given as

$$CS(VI) = \frac{\begin{pmatrix} \alpha^2 n^2\left(\alpha^2 + \alpha + c^2 - \alpha(4-c)c\right) \\ + 2\alpha\gamma^2 nv\left(\alpha(\alpha n + n - 1) - c\left(\alpha^2 n + n - 1\right)\right) + \gamma^4 v^2\left(n\left(\alpha^3 n + n - 2\right) + 1\right) \end{pmatrix}}{2\alpha^3 n^2}.$$

Total surplus is then $W(VI) = CS(VI) + \pi_1(VI) + \pi_2(VI)$.

Welfare implications of vertical integration. The profit of the vertically integrated firm is higher than the profit of the independent publisher. Taking the difference of the consumer surplus in the two cases yields

$$CS(VS) - CS(VI) = \frac{\gamma^2 v\left(v\gamma^2(2n-1) - 2\alpha n(c-\alpha)\right)}{2\alpha^3 n^2} > 0.$$

The above is always positive in the relevant parameter range implying that vertical integration hurts consumer surplus. Thus, we show that in this example consumer's are always worse-off under vertical integration.
Taking the difference of the total surplus in the two cases yields

$$W(VS) - W(VI) = \frac{\gamma^2 v\left(v\gamma^2(2n-1) + v\alpha - 2n\alpha(c-\alpha)\right)}{2\alpha^3 n^2} > 0.$$

The above is always positive in the relevant parameter range. Thus, we show that total welfare falls after vertical integration.

(ii) Case $\bar{u}_1 > \bar{u}_2$. Consumers visit content providers when they receive positive utility. Thus, consumers participate on P_1 and P_2 respectively when

$$V_1 \geq 0 \implies u_1 \geq \bar{u}_1 := c - \gamma q_1 \text{ and } V_2 \geq 0 \implies u_1 \geq \bar{u}_2 := \frac{c-\gamma q_2}{\alpha}.$$

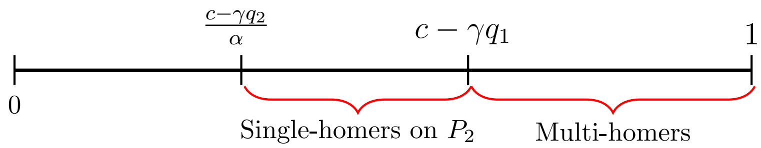

Figure 9: Single and multi-homing consumers.

The associated demands are $D_1 = 0$ and $D_2 = (c-\gamma q_1) - \frac{c-\gamma q_2}{\alpha}$ and $D_{12} = 1-(c-\gamma q_1)$.
Vertical Separation. The equilibrium quality levels at the publisher P_1 and P_2 are respectively given as follows. $q_1(VS) = v\gamma$ and $q_2(VS) = \frac{v\gamma}{\alpha}$ The equilibrium profit of publisher P_1 and P_2 is $\pi_1(VS) = \frac{2v(1-c)+v^2\gamma^2}{2}$ and $\pi_2(VS) = \frac{v(v\gamma^2+2\alpha(\alpha-c))}{2\alpha^2}$. The profit of advertisers and the intermediaries is zero. Consumer surplus in this setting is

$$CS(VS) = \frac{\alpha^2\left(\alpha^2+\alpha+c^2-\alpha(4-c)c\right) + 2\alpha\gamma^2 v\left(\alpha(\alpha+1) - \left(\alpha^2+1\right)c\right) + \left(\alpha^3+1\right)\gamma^4 v^2}{2\alpha^3}.$$

Welfare is given as $W(VS) = CS(VS) + \sum_{p=1,2}\pi_p(VS)$.
Vertical Integration. The equilibrium quality levels at the integrated publisher P_1 and the independent publisher P_2 is respectively given as $q_1(VI) = \frac{v\gamma(n+1)}{n}$, $q_2(VI) = \frac{v\gamma}{\alpha}$.

The profit of the integrated publisher P_1, the independent publisher P_2 and is respectively given as $\pi_1(VI) = \frac{(n+1)v\left(\gamma^2 v(n+1)v+2n(1-c)\right)}{2n^2}$, $\pi_2(VI) = \frac{v\left(\gamma^2 v\left(n\left(n-2\alpha^2\right)-2\alpha^2\right)+2\alpha n(\alpha(n-1)-c(n-\alpha))\right)}{2\alpha^2 n^2}$. Consumers surplus under vertical integration is given as

$$CS(VI) = \frac{\begin{pmatrix} \alpha^2 n^2 \left(\alpha^2 + \alpha + c^2 - \alpha c(4-c)\right) + 2\alpha\gamma^2 nv \left(\alpha(\alpha + \alpha n + n) - c\left(\alpha^2(n+1) + n\right)\right) \\ + \gamma^4 v^2 \left(n^2 + \alpha^3 (n+1)^2\right) \end{pmatrix}}{2\alpha^3 n^2}.$$

Total surplus is then $W(VI) = CS(VI) + \pi_1(VI) + \pi_2(VI)$.

<u>*Welfare implications of vertical integration.*</u> The profit of the vertically integrated firm is higher than the profit of the independent publisher. Taking the difference of the consumer surplus in the two cases yields

$$CS(VS) - CS(VI) = -\frac{\gamma^2 v\left(2(1-c)n + \gamma^2(2n+1)v\right)}{2n^2} < 0.$$

The above expression is always negative in the relevant parameter range implying that vertical integration benefits consumer surplus.

Thus, we show that in this example consumer's are always worse-off under vertical integration.

Taking the difference of the total surplus in the two cases yields

$$W(VS) - W(VI) = \frac{\gamma^2 v\left(v(1-\gamma^2) - 2n(1+\gamma^2 v - c)\right)}{2n^2}.$$

The above is positive if and only if $n > \frac{v(1-\gamma^2)}{2(1+\gamma^2 v - c)}$.

C Model with targeted ads and constant returns to advertising

We provide an alternative version of the model where only some consumers are interested in the product sold by the advertisers, so the platforms perform a matching function between consumers and advertisers. In this version, we relax the assumption of diminishing returns to advertising to the same consumer. We show that the main results of the analysis are robust to this modification.

Assume each consumer is characterized by a type, θ, summarizing a set of characteristics, such as interests, demographics and geographic location, which determine the consumer's relevance to the advertisers. Assume θ is distributed according to a continuous distribution, $Z(\theta)$ with support $[0, \Theta]$. This distribution is independent of the distribution of consumer preferences for content. Assume that for each consumer type there are n advertisers interested in reaching only consumers of that specific type. We assume the advertisers get zero value from reaching any other type of consumer, who are not interested in their product. If the consumer is of the right type, the advertisers get a return of v for each impression sent to the consumer (even if repeated). As in the baseline model, the impression is assigned randomly to the highest bidders if there are two or more such bidders.

Assume intermediary i can observe the type of a consumer visiting publisher p with probability x_p^i. With probability $1-x_p^i$, the intermediary does not observe the consumer's type. Therefore, if the consumer is a multi-homer and the intermediary can track on both publishers, the probability that the intermediary observes her/his type corresponds to the probability that the consumer is identified upon visiting at least one publisher, i.e., $x_{MH}^i = x_1^i(1-x_2^i)+x_2^i(1-x_1^i)+x_1^i x_2^i = x_1^i + x_2^i - (x_1^i x_2^i)$. When auctioning an impression available on the consumer, the intermediary communicates the type to the advertisers, if available. If there is no information available on the consumer, the advertisers are willing to bid zero, because the impression almost surely falls on a consumer who is not of the relevant type.

C.1 Vertical Separation

Consider an advertiser's willingness to pay for an impression on a consumer visiting publisher p. If the consumer is a single-homer, intermediary i identifies her/him with probability x_p^i. If the consumer is a multi-homer, the intermediary identifies her with probability x_{MH}^i. If the consumer is identified, the interested advertisers bid v for the impression. Otherwise, all the advertisers bids zero. Therefore, in a first price auction the price of the impression on intermediary i at stage 3 is $\overline{b}_p^i = v$ if the intermediary identifies the consumer, and zero otherwise. Thus, at stage 4, the bid collected by p for the impression is v if and only if both intermediaries have identified the consumer. That is, $\overline{B}_p = \min(\overline{b}_p^2, \overline{b}_p^1)$ equals v with probability $x_p^1 x_p^2$ if the consumer is a single-homer, $x_{MH}^1 x_{MH}^2$ if the consumer is a multi-homer, and zero otherwise.

Given the above, integrating on all consumer types we can compute the revenue from ads of intermediary $i = 1, 2$ as follows

$$R^i = v\left(\sum_p D_p\left(x_p^i(1-x_p^{-i}) + \frac{x_p^i x_p^{-i}}{2}\right) + D_{YC}\left(2x_{MH}^i(1-x_{MH}^{-i}) + x_{MH}^i x_{MH}^{-i}\right)\right). \quad (22)$$

To interpret this expression, recall that, for each consumer, the intermediary gets to distribute the impression with probability one if and only if it identifies the consumer while the other intermediary does not. If both intermediaries identify, each gets to distribute the impression with probability 1/2.

Consider now how much each intermediary spends to acquire the impressions from the publishers. If the intermediary is the only one to identify the consumer, it pays zero for the impression to the publisher. The intermediary instead pays v if the other intermediary identifies as well. Accordingly, intermediary i's expenditure to acquire the impressions from the two publishers is

$$E^i = v\left(\sum_p D_p\left(\frac{x_p^i x_p^{-i}}{2}\right) + D_{12}\left(x_{MH}^i x_{MH}^{-i}\right)\right), \text{ for } i = 1, 2. \tag{23}$$

Taking the difference between R_i and E_i, we get the intermediary's profit

$$\pi^i = v\left(\sum_p D_p\left(x_p^i(1 - x_p^{-i})\right) + D_{12}\left(2x_{MH}^i(1 - x_{MH}^{-i})\right)\right), \text{ for } i = 1, 2. \tag{24}$$

The above expression shows that, unlike in our baseline model, there is an adtech tax collected by the intermediaries in the VS scenario. The reason is that the intermediaries compete fiercely only to acquire the impressions on consumers that they both identify. Whenever only one intermediary identifies, it is able to obtain the impression at zero price, while still capturing v from the advertisers. Accordingly, the profits earned by each publisher are

$$\pi_p = v\left(D_p x_p^1 x_p^2 + D_{12}\left(x_{MH}^1 x_{MH}^2\right)\right) - k_p(q_p), \text{ for } p = 1, 2. \tag{25}$$

Again, the publisher collects the full value of an impression, v, if and only if the impression is identified by both intermediaries. The value of all other impressions is captured by the intermediaries in the form of the adtech tax.

C.2 Vertical integration

Consider now the scenario where publisher 1 and intermediary 1 are integrated. There are two key differences with respect to the VS scenario. First, only the impressions of publisher 2 are made available to intermediary 2. Second, intermediary 2 cannot track consumers across outlets, so its probability of identifying a consumer is x_2^2 regardless of whether the consumer is a single- or a multi-homer.

The price of the impression auctioned by intermediary i at stage 3 is $\overline{b}_p^i = v$ if the intermediary identifies the consumer, and zero otherwise. Thus, at stage 4, the bid collected by publisher 2 for the impression is v if and only if both intermediaries have identified the consumer. That is, $\overline{B}^2 = \min(\overline{b}_2^2, \overline{b}_2^1)$ equals v with probability $x_2^2 x_2^1$ if the consumer is a single-homer, $x_{MH}^1 x_2^2$ if the consumer is a multi-homer, and zero otherwise. Thus, if the consumer is a multi-homer, the impressions generated on publisher 2 have a lower probability of being identified by both intermediaries, compared to the VS scenario. Recall that, as in the baseline model, an impression is assigned randomly to the highest bidders if there are two or more such bidders.

Given the above, we can compute the revenue from ads of intermediary 2 as follows

$$R_2 = v\left(D_2\left(x_2^2(1-x_2^1)+\frac{x_2^1x_2^2}{2}\right)+D_{12}\left(x_2^2(1-x_{MH}^1)+\frac{x_2^2x_{MH}^1}{2}\right)\right). \tag{26}$$

To interpret this expression, recall that intermediary 2 gets to distribute the impression with probability one if and only if the other intermediary does not identify the consumer. If both intermediaries identify the consumer, each gets to distribute the impression with probability 1/2. Apart from the fact that 2 can only sell impressions on publisher 2, the main difference with (22) is that impressions on multi-homers on publisher 2 are identified by intermediary 2 with lower probability. Intermediary 2's expenditure to acquire the impressions from publisher 2 is

$$E_2 = v\left(D_2\left(\frac{x_2^1x_2^2}{2}\right)+D_{12}\left(\frac{x_{MH}^1x_2^2}{2}\right)\right). \tag{27}$$

Taking the difference between R_2 and E_2, we get the intermediary's profit

$$\pi^2 = v\left(D_2\left(x_2^2(1-x_2^1)\right)+D_{12}\left(x_2^2(1-x_{MH}^1)\right)\right). \tag{28}$$

Observe that there is still is an adtech tax collected by the intermediary, but this tax is smaller than in the separation scenario.

We can in a similar way write the revenue of the integrated firm:

$$R^1 = v\left(D_2\left(x_2^1(1-x_2^2)+\frac{x_2^1x_2^2}{2}\right)+D_1x_1^1\right)+$$
$$+vD_{12}\left(x_{MH}^1(1-x_2^2)+\frac{x_2^2x_{MH}^1}{2}+x_{MH}^1\right).$$

Firm 1 retains not only all the revenue from impressions on publisher 1, but captures also some of the revenue from the impressions on 2. The expenditure for acquiring the latter impressions is

$$E^1 = v\left(D_2\left(\frac{x_2^1x_2^2}{2}\right)+D_{12}\left(\frac{x_{MH}^1x_2^2}{2}\right)\right), \tag{29}$$

so that the net profit of firm 1 is

$$\pi^1 = v\left(D_2\left(x_2^1(1-x_2^2)\right)+D_1x_1^1+D_{12}\left(x_{MH}^1(1-x_2^2)+x_{MH}^1\right)\right)-k_1(q_1), \tag{30}$$

which can be conveniently be rewritten as

$$\begin{aligned}\pi^1 = v\left((D_1+D_{12})x_1^1x_2^1+D_1x_1^1(1-x_2^1)+D_2x_2^1(1-x_2^2)\right)+\\ +v\left(D_{12}\left(x_{MH}^1(1-x_2^2)+x_{MH}^1-x_1^1x_1^2\right)\right)-k_1(q_1).\end{aligned} \tag{31}$$

Finally, we can write the profit of publisher 2 as

$$\begin{aligned}\pi_2 = v\left(D_2 x_2^2 x_2^1 + D_{12} x_2^2 x_{MH}^1\right) - k_2(q_2) = \\ v\left((D_2 + D_{12}) x_2^2 x_2^1 + D_{12}\left(x_2^2 (x_{MH}^1 - x_2^1)\right)\right) - k_2(q_2).\end{aligned} \tag{32}$$

By comparing the profits of the intermediaries in the two scenarios, we can establish that the adtech tax collected on publisher 2's impressions by the integrated firm increases, contrary to the tax collected by intermediary 2.

Overall, however, the total adtech tax collected by intermediaries on publisher 2 increases under VI compared to the VS scenario, which is consistent with Proposition 1. This is in particular due to the impressions on multi-homers. The share of impressions identified uniquely by the intermediary 1 in the VS scenario is $x_{MH}^1(1 - x_{MH}^2)$ and it increases to $x_{MH}^1(1 - x_2^2)$ under VI. On the contrary, the share of impressions on multi-homers uniquely identified by 2 is $x_{MH}^2(1-x_{MH}^1)$ under VS and decreases to $x_2^2(1-x_{MH}^1)$ under VI. Finally, the share of impressions identified by both intermediaries decreases from $x_{MH}^2 x_{MH}^1$ under VS to $x_2^2 x_{MH}^1$. As a result, the profit of publisher 2 is lower than in the VS scenario (all else given), because the publisher only retains the revenue from impressions that are identified by both intermediaries. Under VI, the probability that this occurs is lower because intermediary 2 cannot track consumers across publishers. Hence, the overall volume of impressions on identified consumers goes down. Moreover, a larger share of these impressions is identified by a single intermediary (the integrated one), so publisher 2 does not obtain any revenue from them.

C.3 Choice of quality investment

Consider now the choice of quality investment at stage 1. In the VS scenario, starting from (25), the pair of equilibrium quality levels, $(q_1(VS), q_2(VS))$ satisfies the following expressions:

$$\frac{\partial \pi_p^{VS}}{\partial q_p} = v\left(\left(\frac{\partial D_p}{\partial q_p} + \frac{\partial D_{12}}{\partial q_p}\right) x_p^1 x_p^2 + \frac{\partial D_{12}}{\partial q_p}\left(x_{MH}^1 x_{MH}^2 - x_p^1 x_p^2\right)\right) - \frac{\partial k_p(q_p)}{\partial q_p} = 0, \quad p = 1, 2.$$

In the VI scenario, starting from (31) and (32), the pair of equilibrium quality levels, $(q_1(VI), q_2(VI))$ satisfies the following

$$\begin{aligned}\frac{\partial \pi_1^{VI}}{\partial q_1} = v\left(\left(\frac{\partial D_1}{\partial q_1} + \frac{\partial D_{12}}{\partial q_1}\right) x_1^1 x_1^2 + \frac{\partial D_1}{\partial q_1} x_1^1(1 - x_1^2) + \frac{\partial D_2}{\partial q_1} x_2^1(1 - x_2^2)\right) + \\ v\left(\frac{\partial D_{12}}{\partial q_1}\left(x_{MH}^1 + x_{MH}^1(1 - x_2^2) - x_1^1 x_1^2\right)\right) - \frac{\partial k_1(q_1)}{\partial q_1} = 0,\end{aligned}$$

$$\frac{\partial \pi_2^{VI}}{\partial q_2} = v\left(\left(\frac{\partial D_2}{\partial q_2} + \frac{\partial D_{12}}{\partial q_2}\right) x_2^1 x_2^2 + \frac{\partial D_{12}}{\partial q_2}(x_2^2(x_{MH}^1 - x_2^1)\right) - \frac{\partial k_2(q_2)}{\partial q_1} = 0.$$

Comparing these expressions, and recalling that $\frac{\partial D_2}{\partial q_1} = -\frac{\partial D_{12}}{\partial q_1}$, we can establish that $\frac{\partial \pi_1^{VI}}{\partial q_1} > \frac{\partial \pi_1^{VS}}{\partial q_1}$ and $\frac{\partial \pi_2^{VI}}{\partial q_2} < \frac{\partial \pi_2^{VS}}{\partial q_2}$, so the same comparison as in Proposition 2 holds.